



FIG. 1. (Color) Mixing in a von Kármán vortex street. For details see text.

## Lagrangian feature extraction of the cylinder wake

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We extract Lagrangian features of the two-dimensional von Kármán vortex street behind a circular cylinder. The temporal convergence and separation of particles is monitored over two shedding periods. This behavior is quantified by the distance of nearby particles during this time window. The finite time Lyapunov exponent (FTLE)<sup>1</sup> measures the logarithm of the maximum distance of these nearby particles. We compute the FTLE based on the first variational form,<sup>2</sup> i.e., integrating a locally linear flow.

In Fig. 1, red coloring indicates regions of particle divergence in forward time. Blue regions show convergence (divergence in backward time). The height of the gray surface represents the maximum of both corresponding FTLE val-

ues. The direction of the flow field is indicated by lines on the surface.

The intersections of both curves mark high separation on the one side and high convergence on the other side. These points are interpreted as a Lagrangian version of saddle points from topology.<sup>3</sup> Furthermore, the ridges of the FTLE build attractive and separating invariant manifold “arms” that mark domains of particle attraction and separation. Thus, the mixing of the von Kármán vortex street is characterized by the Lagrangian saddle points and the corresponding domains.

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