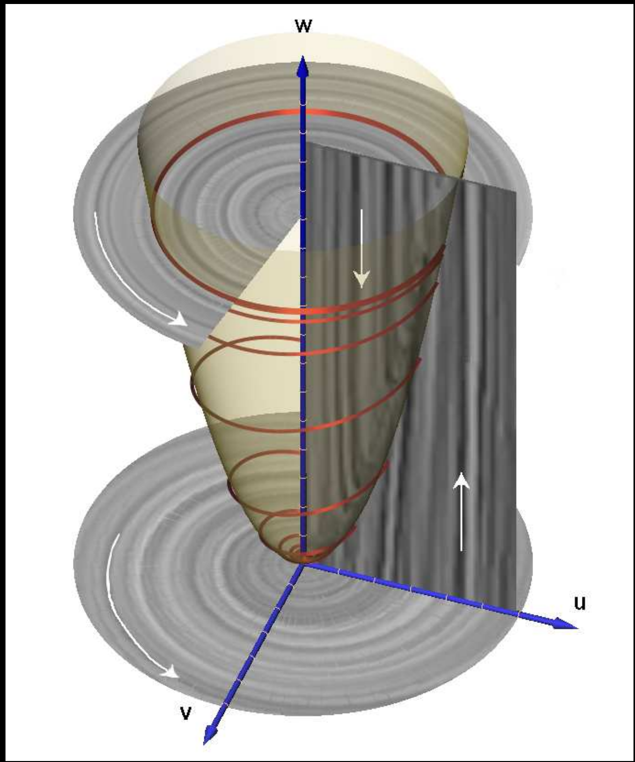
Low-dimensional modelling
— Towards attractor control *I*

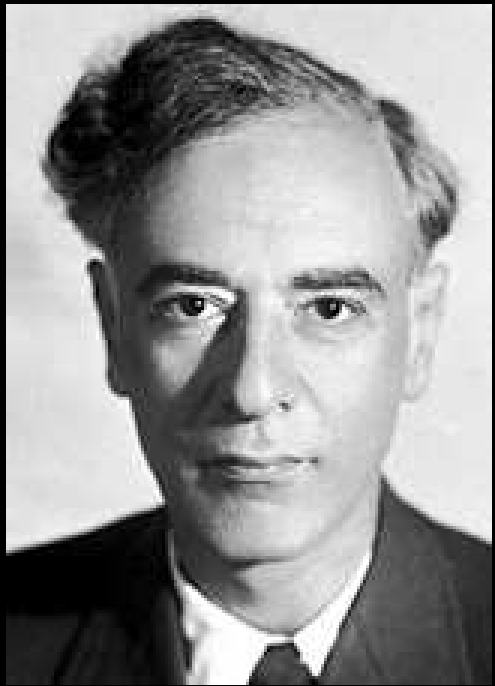


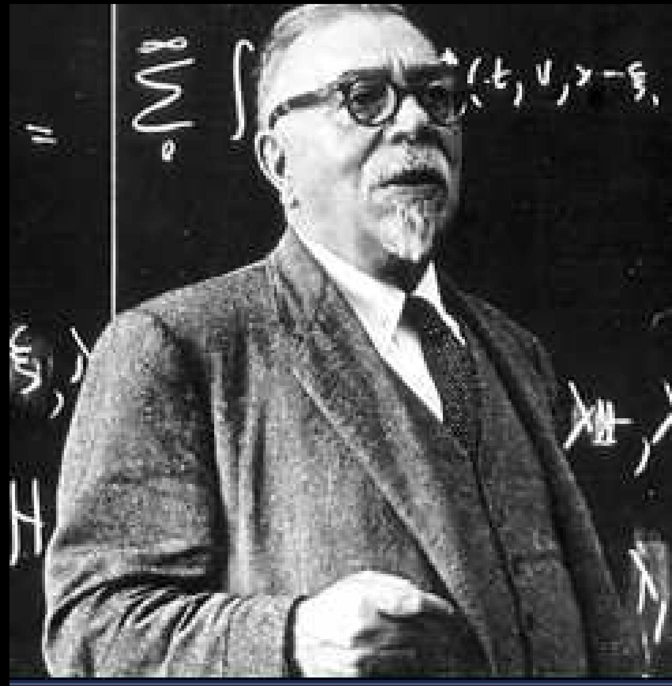
Bernd R. Noack & friends

Berlin Institute of Technology & *elsewhere*

CISM course "Reduced-Order Modelling for Flow Control", Udine, September 15–19, 2008

Low-dimensional modelling
— Towards attractor control





Bernd R. Noack & friends

Berlin Institute of Technology & *elsewhere*

CISM course "Reduced-Order Modelling for Flow Control", Udine, September 15–19, 2008

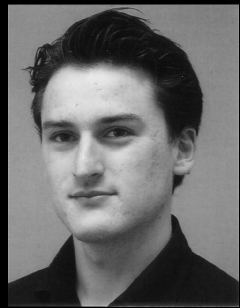
Bernd's friends

— subset for FTT ordered by distance from my office

at the Berlin Institute of Technology

Michael Schlegel

Mark Luchtenburg



Mark Pastoor

Rudibert King

and elsewhere

Gerd Mutschke

Marek Morzyński

Pierre Comte

Gilead Tadmor

Boye Ahlborn

My lectures

1 (Mo) Motivation of Galerkin method, 2 examples

2 (Tu) Empirical Galerkin method based on POD

3 (Tu) POD-based Galerkin models of natural flow

4+5 **Towards a attractor control**

(Th) **Purpose of this lecture:**

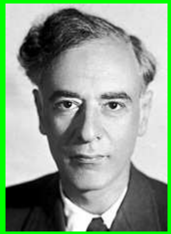
- Make you excited about Galerkin modelling
- by connecting to (non)linear dynamics, statistical physics and cybernetics
- and by sketching a flow control of the future.

Overview

1. Introduction

. . . . physics & cybernetics dreams revisited

2. Mean-field modelling

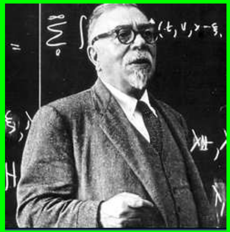


. . . . complete order / stability theory

3. Attractor modelling

. . . . complete disorder / statistical physics

4. Attractor control



. . . . Maxwellian and other deamons

5. Summary and outlook

Overview

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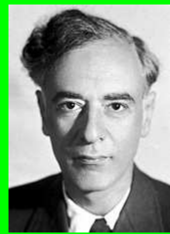

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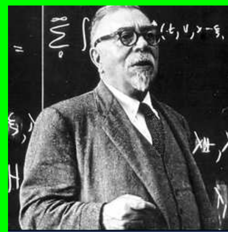
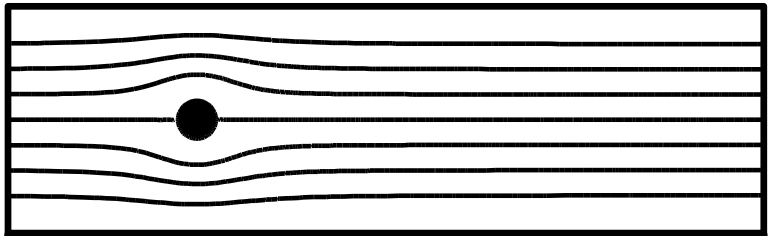


5. Summary and outlook

Phenomenogram of cylinder wake

Reynolds number $Re = \frac{UD}{\nu}$

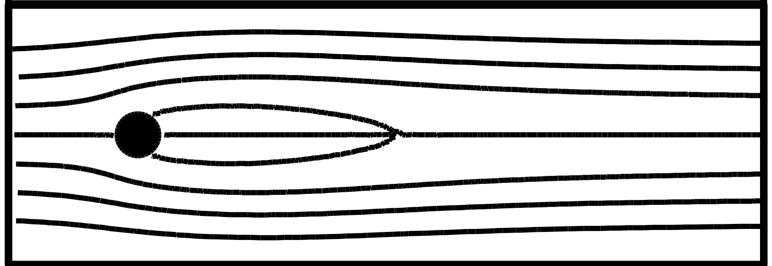
$Re < 4$



2D steady flow

without vortex pair

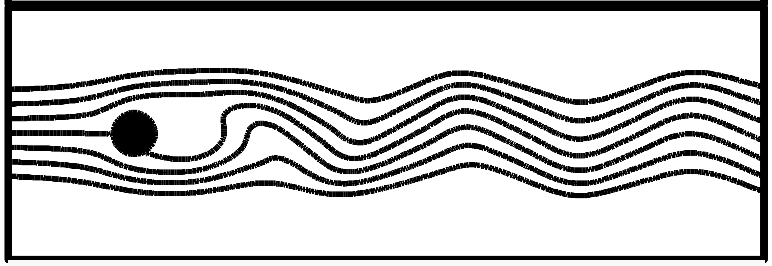
$Re < 47$



2D steady flow

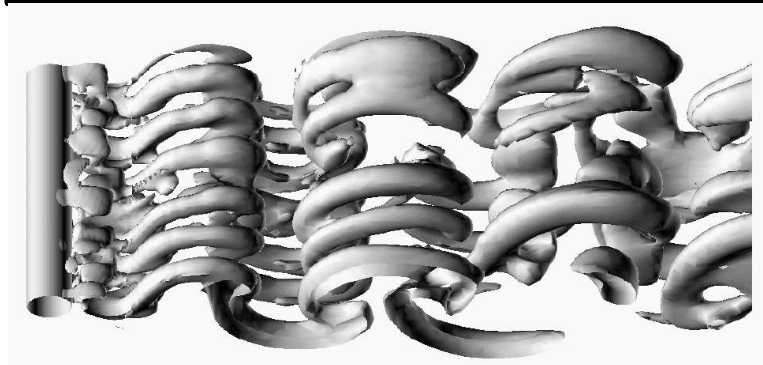
with vortex pair

$Re < 180$



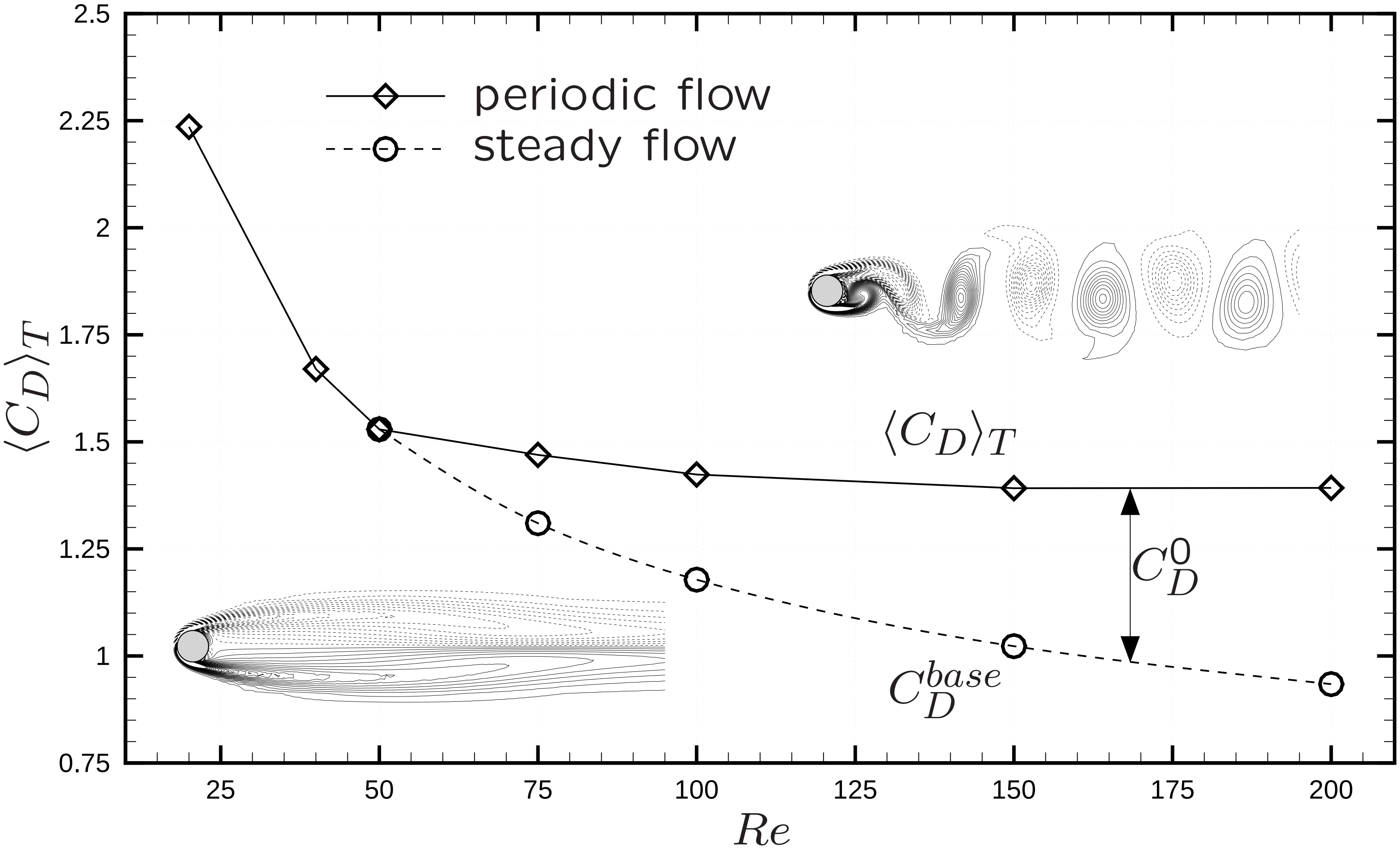
2D vortex shedding

$180 < Re$



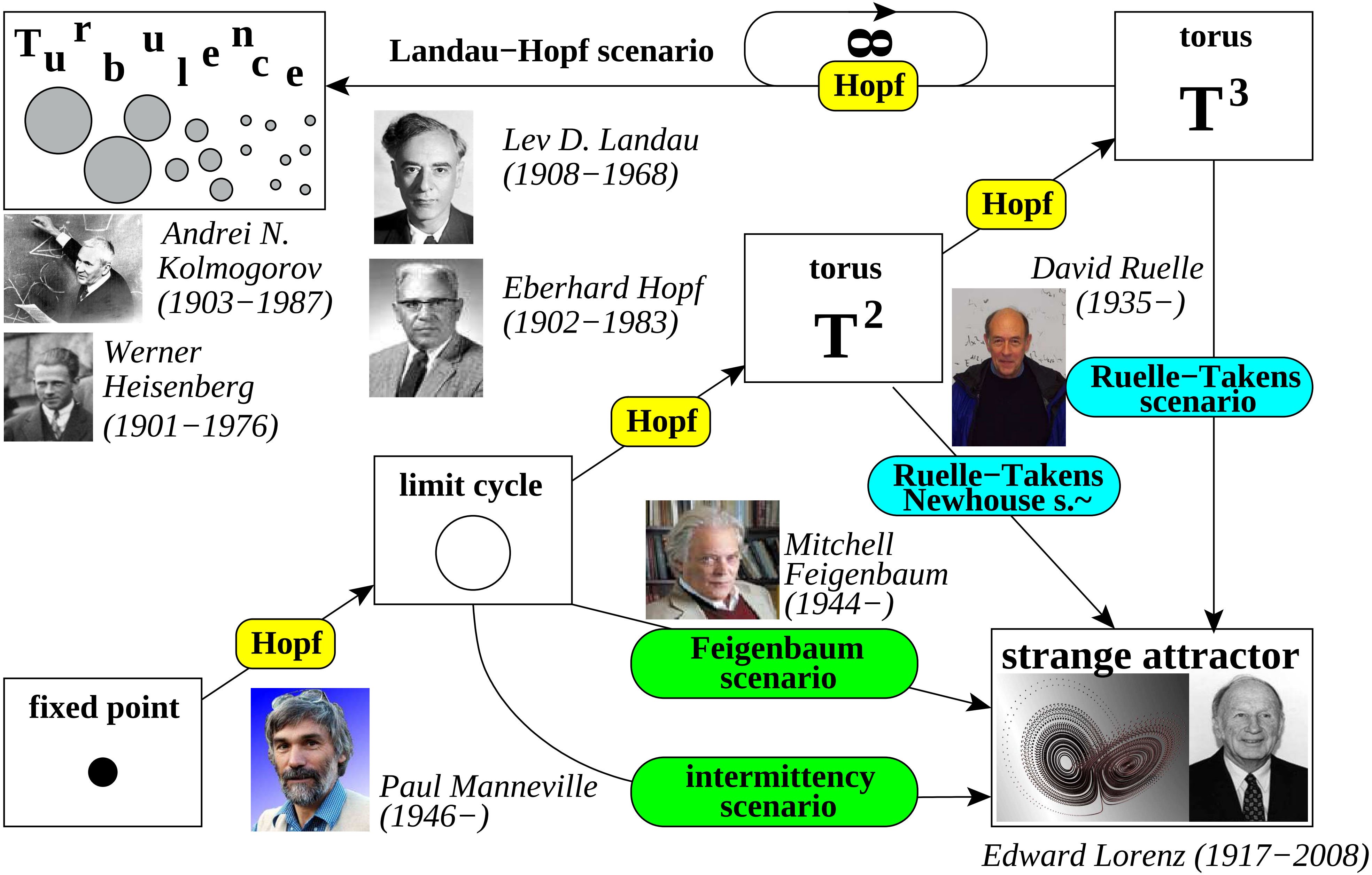
2D vortex shedding superimposed by 3D modes / fluctuations

Drag of cylinder wake



From Protas & Wesfreid *2002 Phys. Fluids*

Dream #1: Instabilities $\mapsto$ turbulence



Dream #2: Statistical physics $\mapsto$ turbulence

Ludwig Boltzmann
(1840–1906)

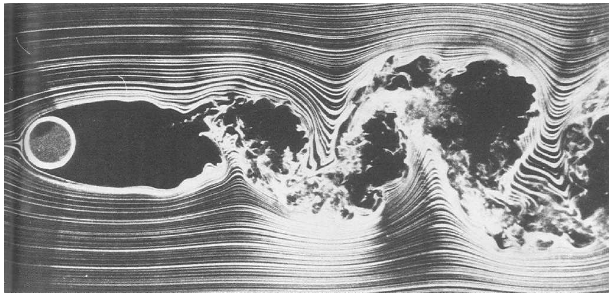
Equivalent subsystems:
1877: Entropy

$S = k \, \ln W$

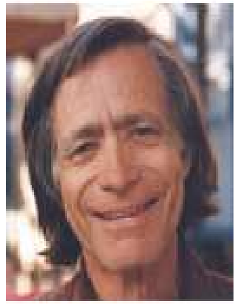
Lars Onsager
(1903–1976)

Particle/vortex picture:
1949: point vortices in 2D flows
= thermodyn. degree of freedom

Hans W Liepmann (1914-)'s
WARNING:
Don't forget →



Robert H Kraichnan
(1928–2008)



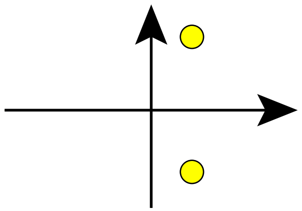
Wave/Galerkin picture:
1955: Fourier modes
= thermodyn. degrees of freedom
(absolute equilibrium ensemble)

How to partition the flow in equivalent subsystems (atoms)
(= thermodynamic degrees of freedom)???

Dream #3: Control $\mapsto$ turbulence

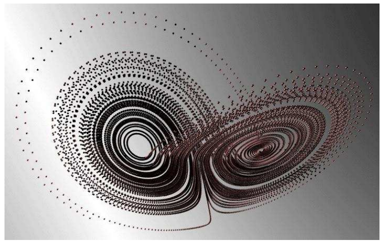
linear dynamics

da/dt = A a + B b



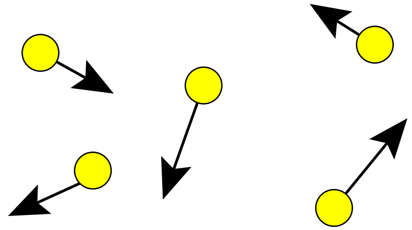
strange attractor

da/dt = f (a,b)



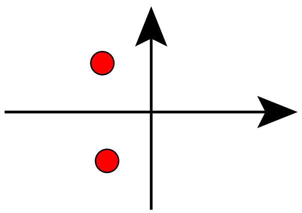
statistical physics

S = k ln W



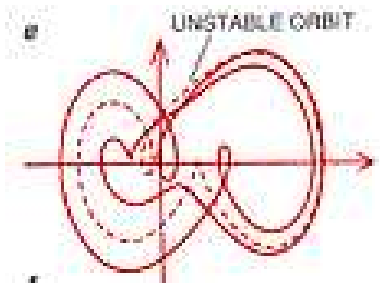
linear control

b =K a



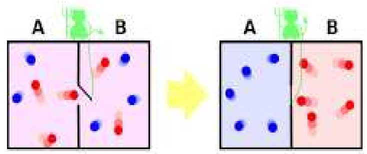
Anno
Dazumal

chaos control



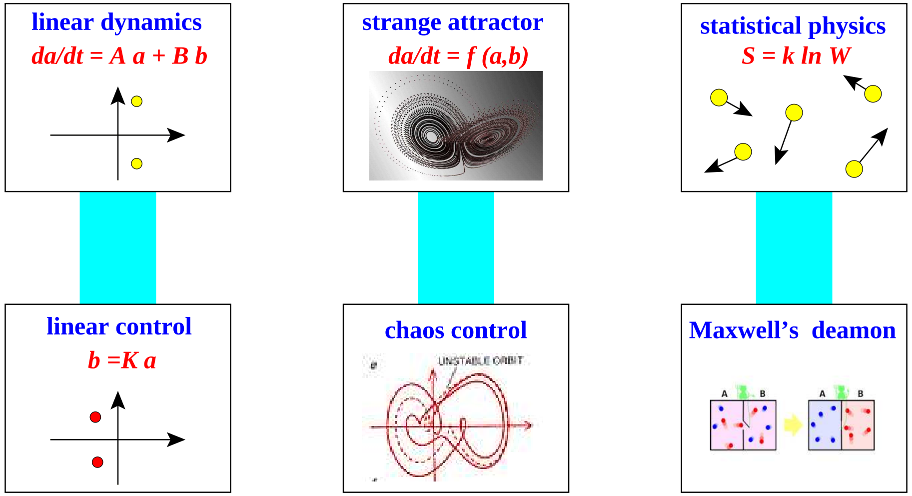
Ott, Grebogi, Yorke
1990 PRL

Maxwell's deamon



Maxwell 1867
Wiener 1948

Turbulence control $\mapsto$ transport vehicles

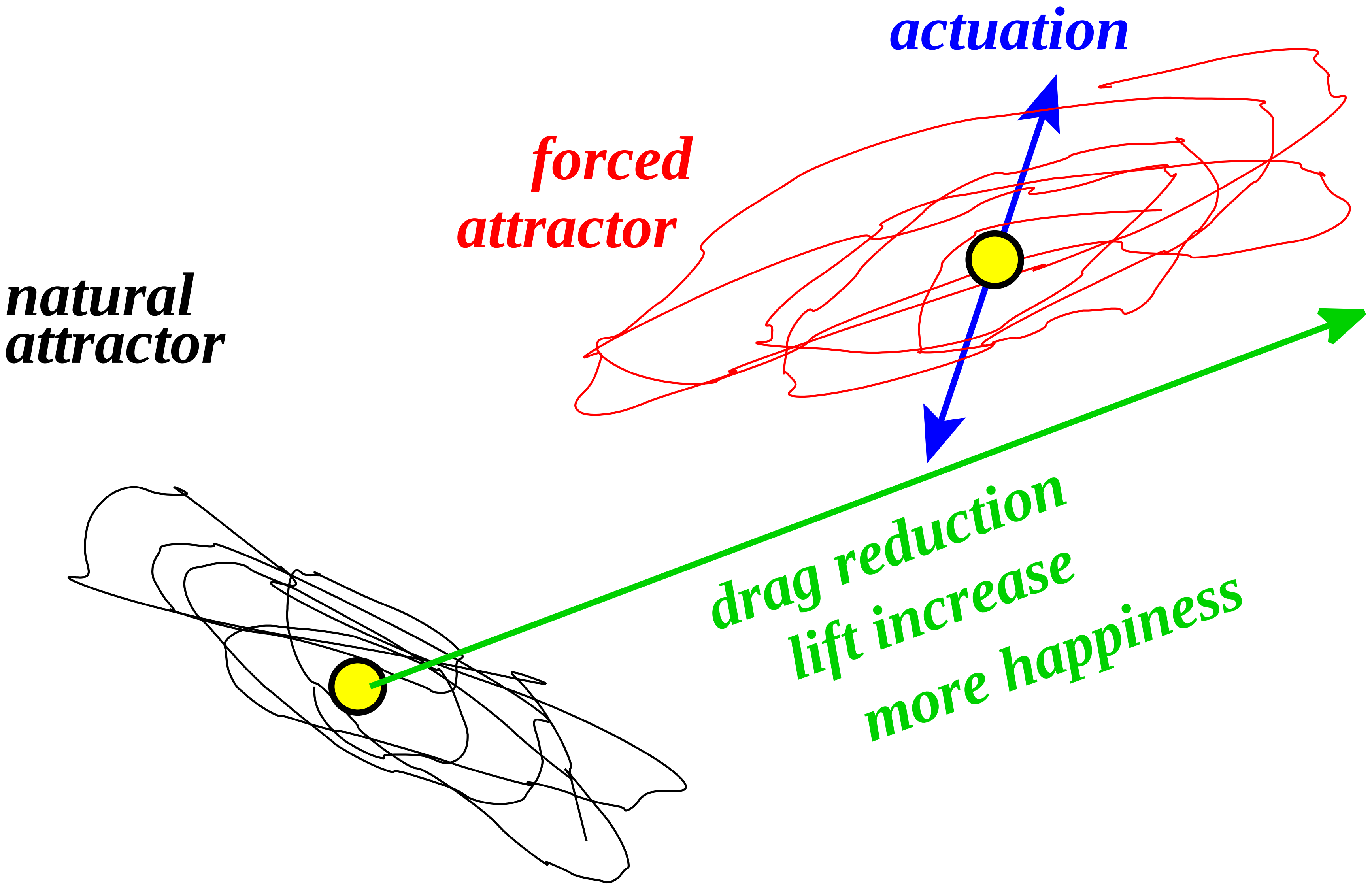


Anno Dazumal — *Ott, Grebogi, Yorke 1990 PRL* — *Maxwell 1867 Wiener 1948*

Promising modelling and control methods for practical applications?

Turbulence control = attractor control

Phase space



Overview

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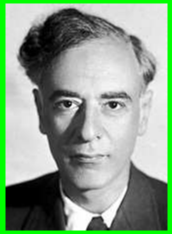
   

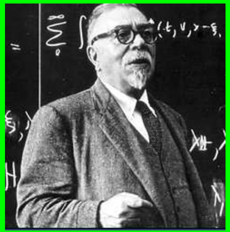
   . . . . *complete order / stability theory*

3. **Attractor modelling**

   . . . . *complete disorder / statistical physics*

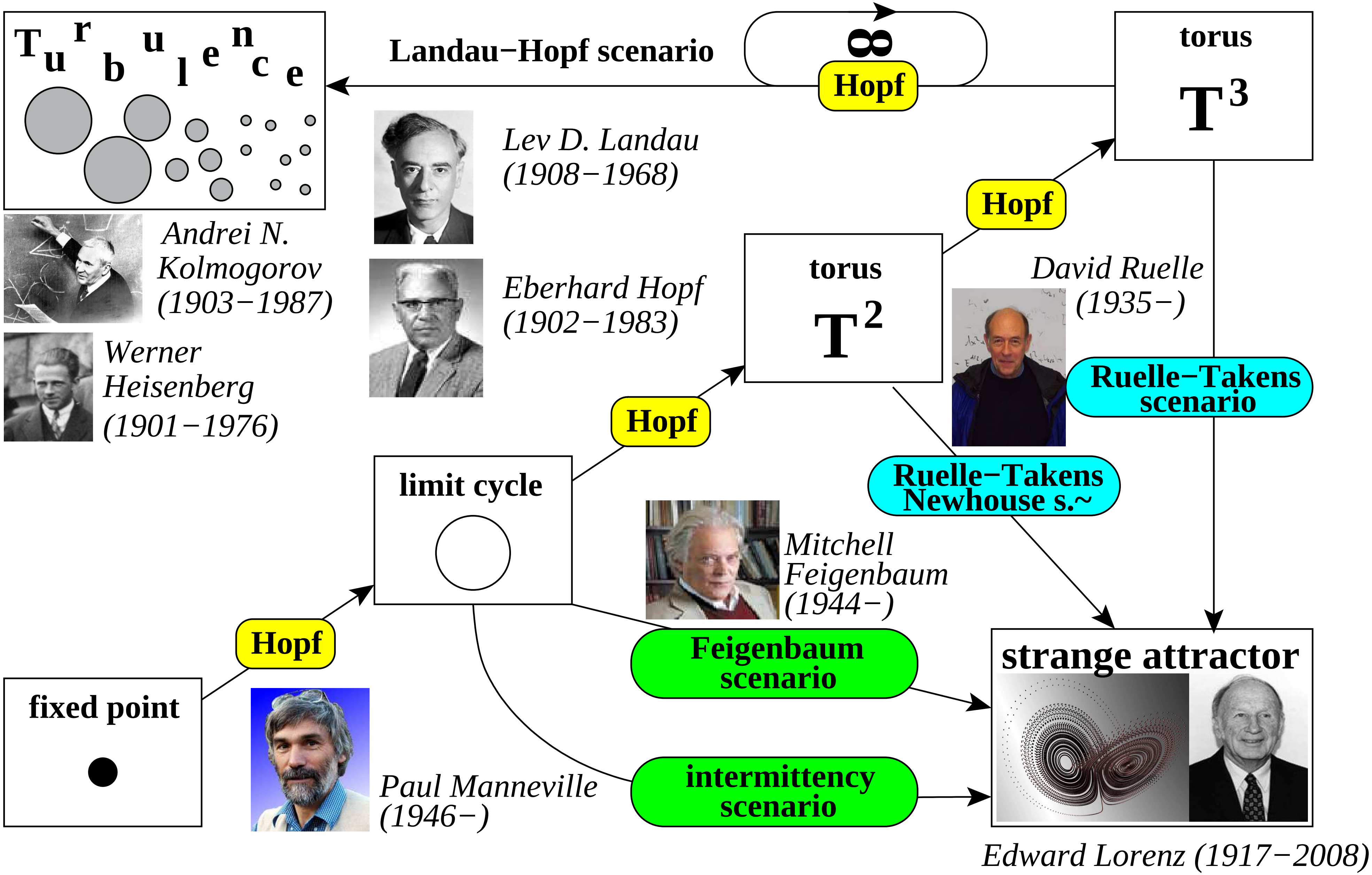
4. **Attractor control**

   . . . . *Maxwellian and other deamons*

5. **Summary and outlook**

Dream #1: Instabilities $\mapsto$ turbulence

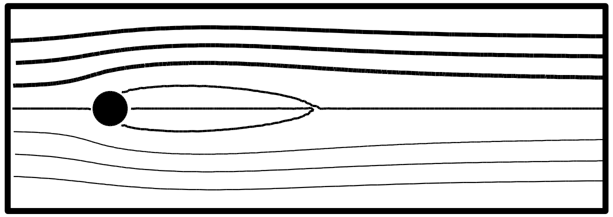


Global linear stability analysis

— Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —

(Unstable) steady Navier-Stokes solution at $Re = 100$

$$\nabla \cdot (\mathbf{u}_s \mathbf{u}_s) = -\nabla p_s + \nu \triangle \mathbf{u}_s$$



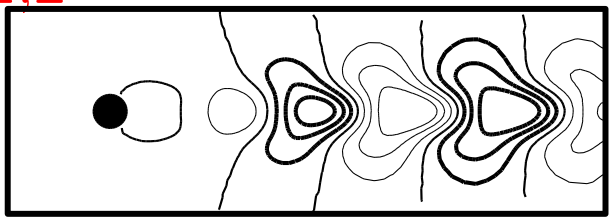
Linearized Navier-Stokes equation $\mathbf{u} = \mathbf{u}_s + \mathbf{u}'$

$$\partial_t \mathbf{u}' + \nabla \cdot \left(\mathbf{u}_s\, \mathbf{u}' + \mathbf{u}'\, \mathbf{u}_s \right) = -\nabla p' + \nu \triangle \mathbf{u}'$$

Instability described by normal mode $\lambda_{1,2} = 0.139 \pm \imath 0.855$

$$\mathbf{u}' \sim e^{\lambda_{1,2} t}\ \mathbf{f}_1(\mathbf{x})$$

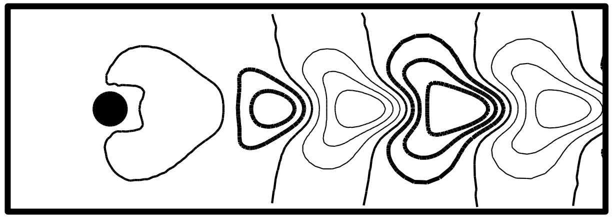
$$\mathbf{u}_1 = \Re\{\mathbf{f}_1\}$$



Galerkin approximation

$$\mathbf{u} = \mathbf{u}_s + a_1 \mathbf{u}_1 + a_2 \mathbf{u}_2$$

$$\mathbf{u}_2 = \Im\{\mathbf{f}_1\}$$

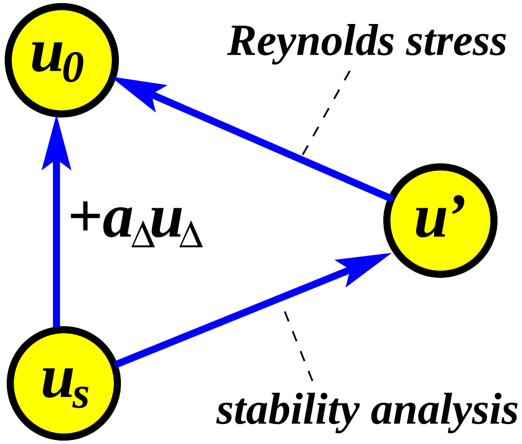


- **2-dim. Galerkin model reproduces instability.**

Mean-field model

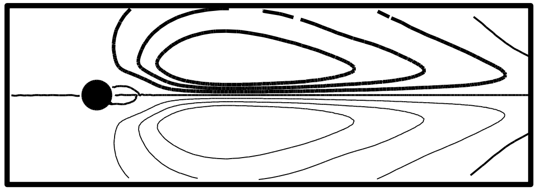
≡ *Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM*

Mean-field damping mechanism

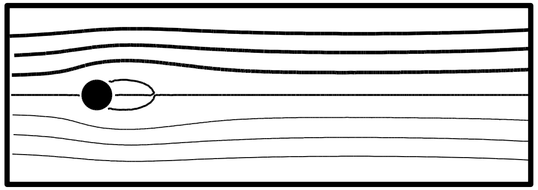


≡ Stuart (1958)

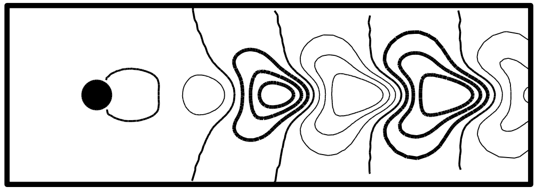
shift mode $\mathbf{u}_\Delta$



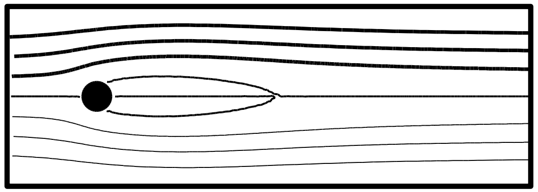
mean flow $\mathbf{u}_0$



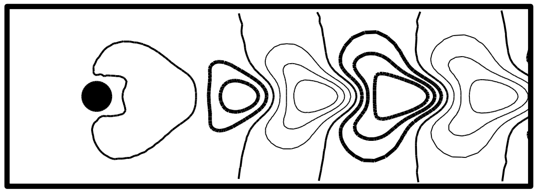
eigenmode $\mathbf{u}_1$



steady solution $\mathbf{u}_s$



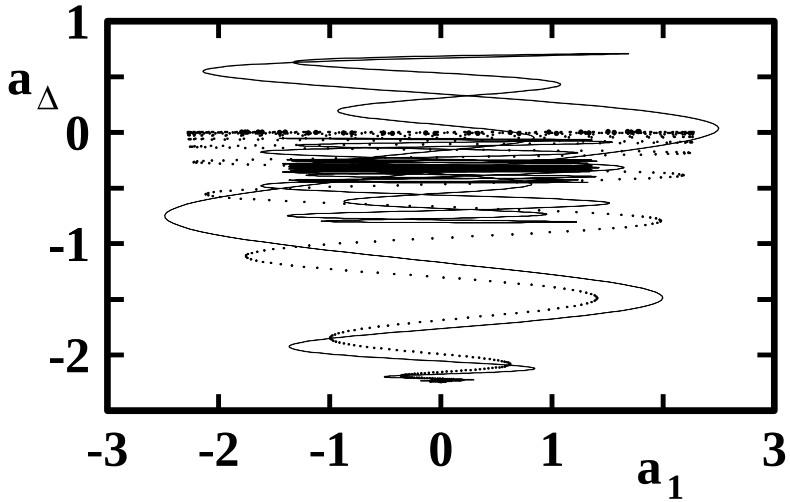
eigenmode $\mathbf{u}_2$



Galerkin approximation

$$\mathbf{u} = \mathbf{u}_0 + a_\triangle \mathbf{u}_\triangle + a_1 \mathbf{u}_1 + a_2 \mathbf{u}_2$$

Galerkin solution



Fluctuation amplitude

45% too low!

Shift mode prevents divergence.

Generalized mean-field model

— ≡ Noack, Afanasiev, Morzyński, Thiele & Tadmor 2003 JFM —

Galerkin model

(1a) $\mathbf{u} = \mathbf{u}_s + \sum_{i=1}^{3} a_i\, \mathbf{u}_i$

(1b) $\frac{da_i}{dt} = \sum_{j=0}^{3} l_{ij} a_j + \sum_{j,k=0}^{3} q_{ijk} a_j\, a_k$

$\mathbf{u}_s$: steady flow
$\mathbf{u}_{1,2}$: oscillatory modes
$\mathbf{u}_3$: shift mode

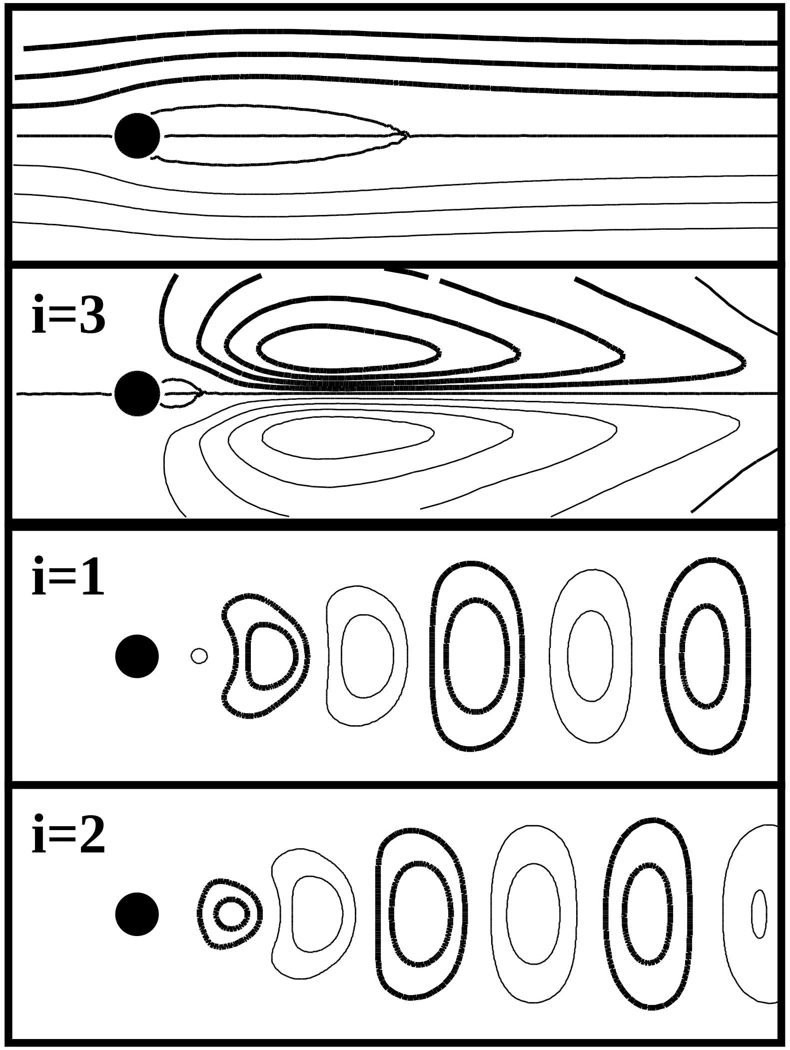
Polar coordinates

$a_1 = A \cos\phi,\ a_2 = A \cos\phi$

(2a) $\frac{dA}{dt} = (\sigma - \beta a_3)\, A +$ *h.h.*

(2b) $\frac{d\phi}{dt} = (\omega + \gamma a_3) +$ *h.h.*

(2c) $\frac{da_3}{dt} = (\sigma_3 a_3 + c A^2) +$ *h.h.*



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(1b) $\frac{da_i}{dt} = \sum_{j=0}^{3} l_{ij} a_j + \sum_{j,k=0}^{3} q_{ijk} a_j \, a_k$

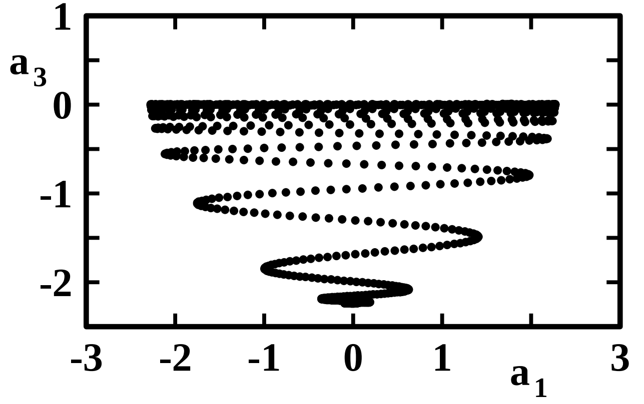
$\mathbf{u}_s$: steady flow
$\mathbf{u}_{1,2}$: oscillatory modes
$\mathbf{u}_3$: shift mode

Centre manifold

Using $|\sigma_3| \gg \sigma$ (2c) becomes:

$0 = (\sigma_3 a_3 + c A^2) \Rightarrow$

(3) $a_3 = -\frac{c}{\sigma_3} A^2$



Polar coordinates

$a_1 = A \cos\phi, \; a_2 = A \cos\phi$

(2a) $\frac{dA}{dt} = (\sigma - \beta a_3) \, A +$ h.h.

(2b) $\frac{d\phi}{dt} = (\omega + \gamma a_3) +$ h.h.

(2c) $\frac{da_3}{dt} = (\sigma_3 a_3 + c A^2) +$ h.h.

Landau equations

(3) in (2a,b) yields

(4a) $\frac{dA}{dt} = \sigma A - \beta^\star A^3$

(4b) $\frac{d\phi}{dt} = \omega \quad + \gamma^\star A^2$

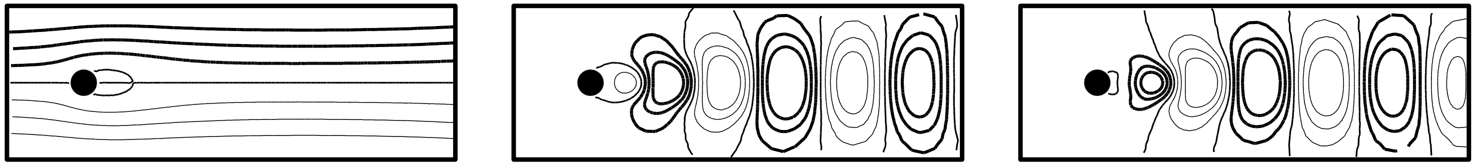
$\sigma, \omega, \beta^\star, \gamma^\star > 0$

Transient dynamics of wake

Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —

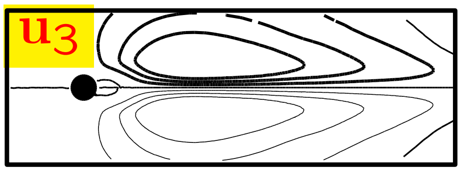
Operating condition II

on attractor



↑

transient on paraboloid



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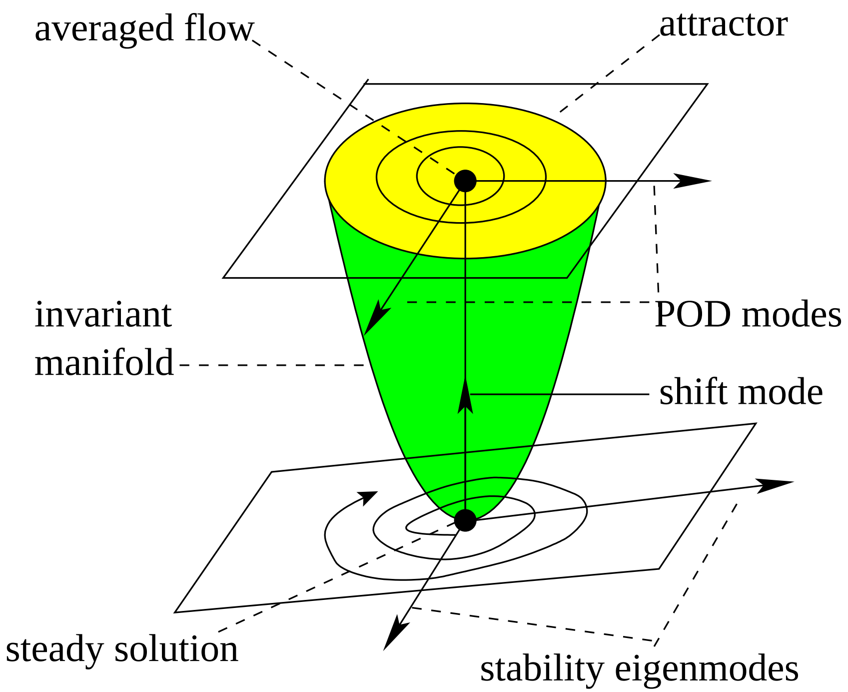


$$\frac{d}{dt}a_1 = \sigma a_1 - \omega a_2$$
$$\frac{d}{dt}a_2 = \sigma a_2 + \omega a_1$$
$$\frac{d}{dt}a_3 = -\sigma_3 a_3 + c A^2$$

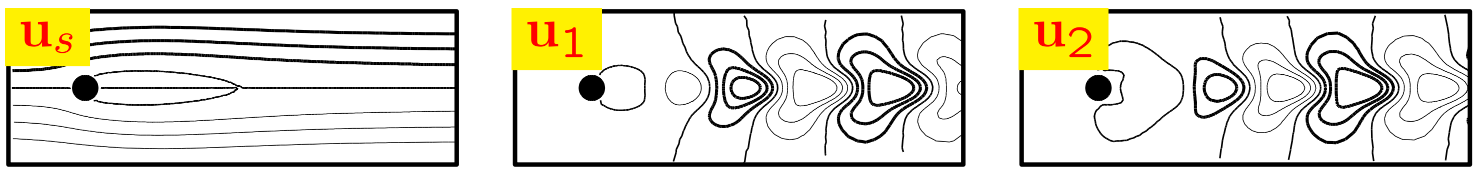
$$\sigma = \sigma_1 - \beta a_3$$
$$\omega = \omega_1 + \gamma a_3$$
$$A^2 = a_1^2 + a_2^2$$

Landau equation

$$\frac{d}{dt}A = \sigma_1 A - \beta^\star A^3$$

Operating condition I

near fixed point



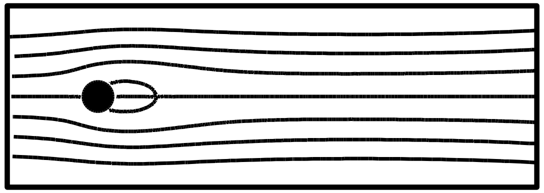
POD Galerkin model

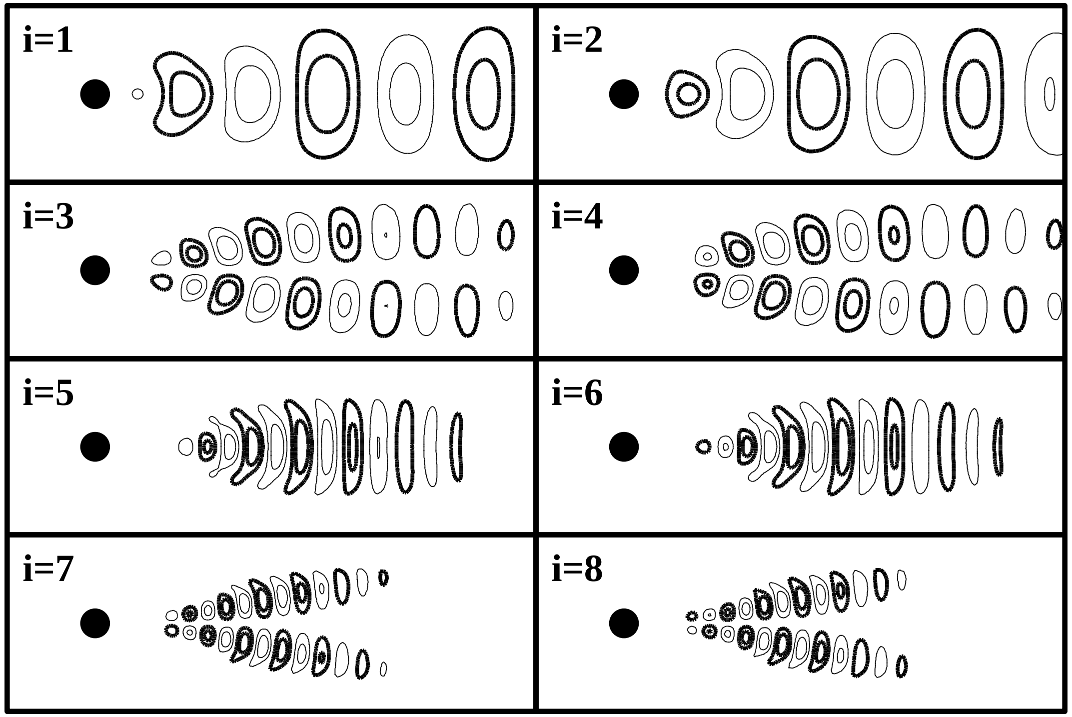
Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM

POD at $Re = 100$

Deane et al (1991) PF

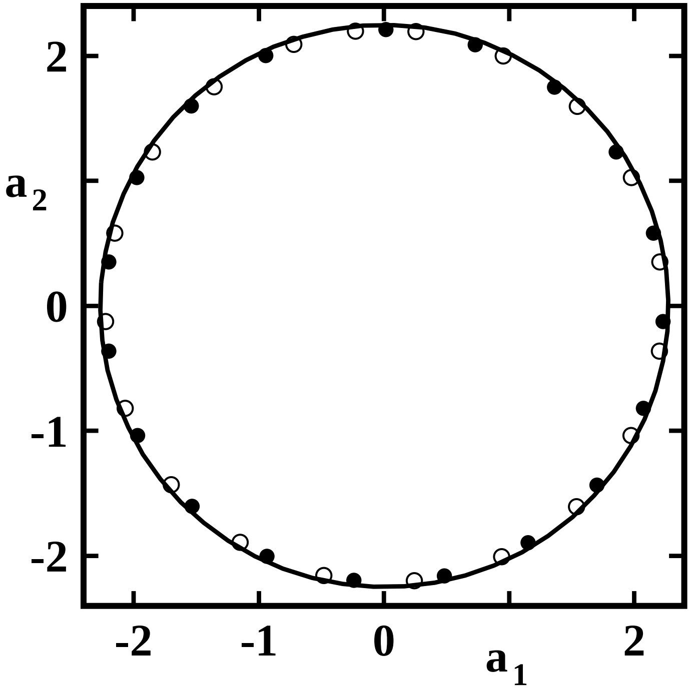
$$\mathbf{u} = \sum_{i=0}^{8} a_i \mathbf{u}_i$$





Galerkin solution

$$\frac{da_i}{dt} = \nu \sum_j l_{ij} a_j + \sum_{j,k} q_{ij} a_j a_k$$



- 8-dim. POD model reproduces DNS.

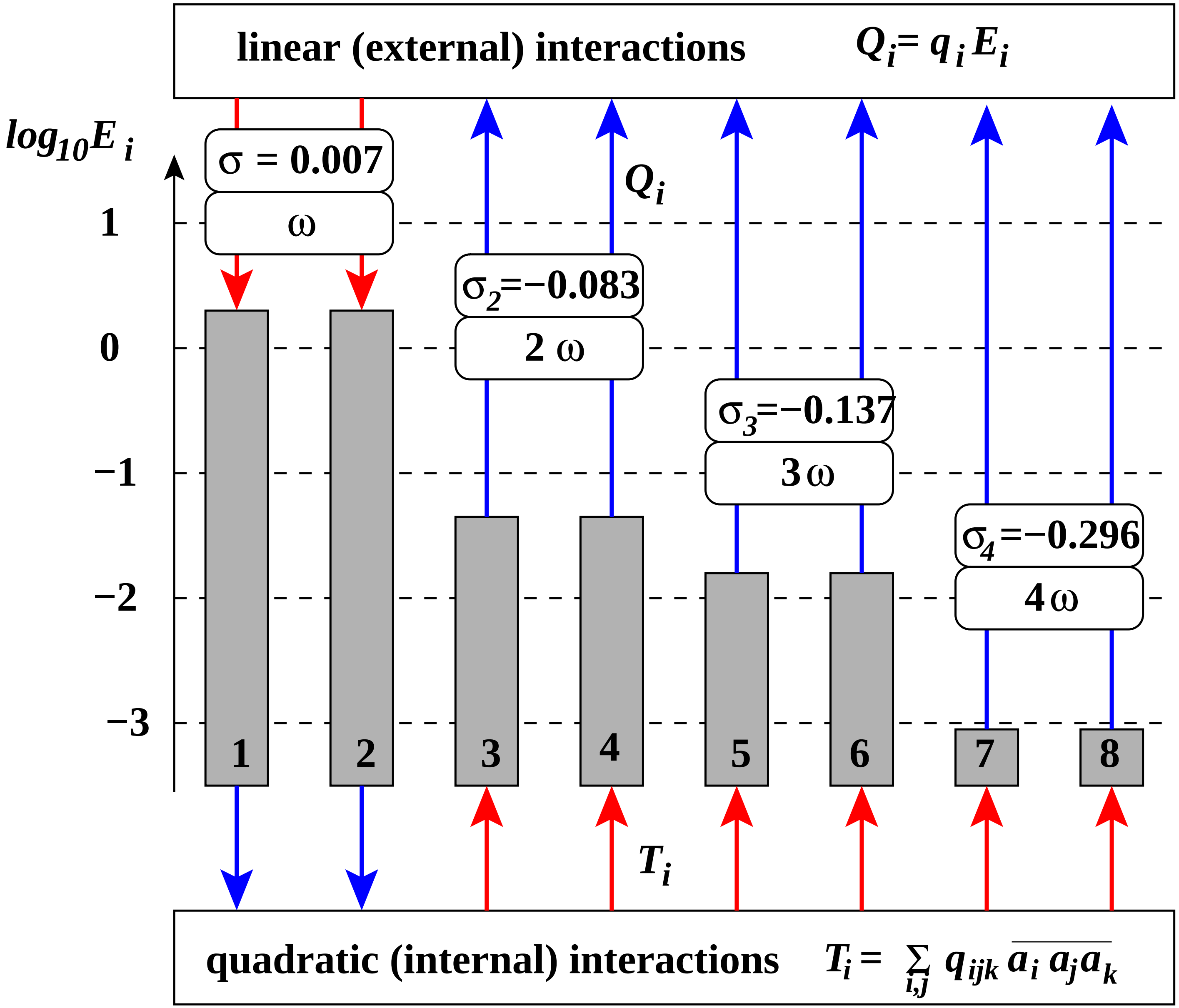
POD Galerkin system of cylinder wake II

— Noack, Afanasiev, Morzyński, Thiele & Tadmor 2003 JFM —

Modal energetics:

$0 = Q_i + T_i$

$Q_i = q_i E_i$



Structural instability of POD GM — *Part I*

— *Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM* —

3-dim. SODE ($\mu = 0.1$)

$$\frac{du}{dt} = (\mu - w)\, u - v$$

$$\frac{dv}{dt} = (\mu - w)\, v + u$$

$$\frac{dw}{dt} = -w + (u^2 + v^2)$$

Unstable fixed point:

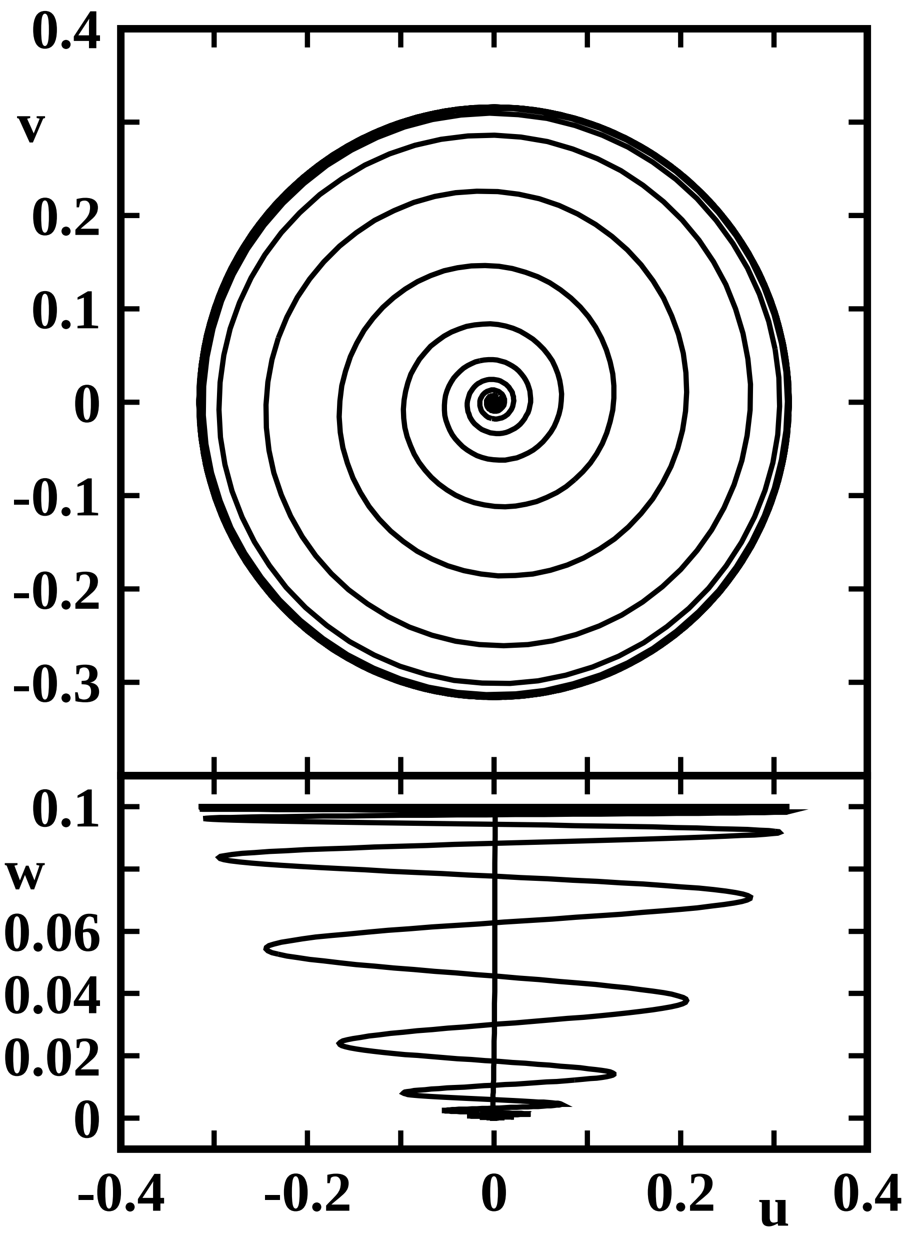
$$u = v = w = 0$$

Stable limit cycle:

$$u = \sqrt{\mu}\cos t$$

$$v = \sqrt{\mu}\sin t$$

$$w = \mu$$



Structural instability of POD GM — *Part II*

— *Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM* —

POD Galerkin approximation: $\mathbf{u} = \mathbf{u}_0 + a_1\mathbf{u}_1 + a_2\mathbf{u}_2$

where $\mathbf{u}_0 = (0, 0, \mu^{\text{Ref}})$, $\mathbf{u}_1 = (1, 0, 0)$, $\mathbf{u}_2 = (0, 1, 0)$

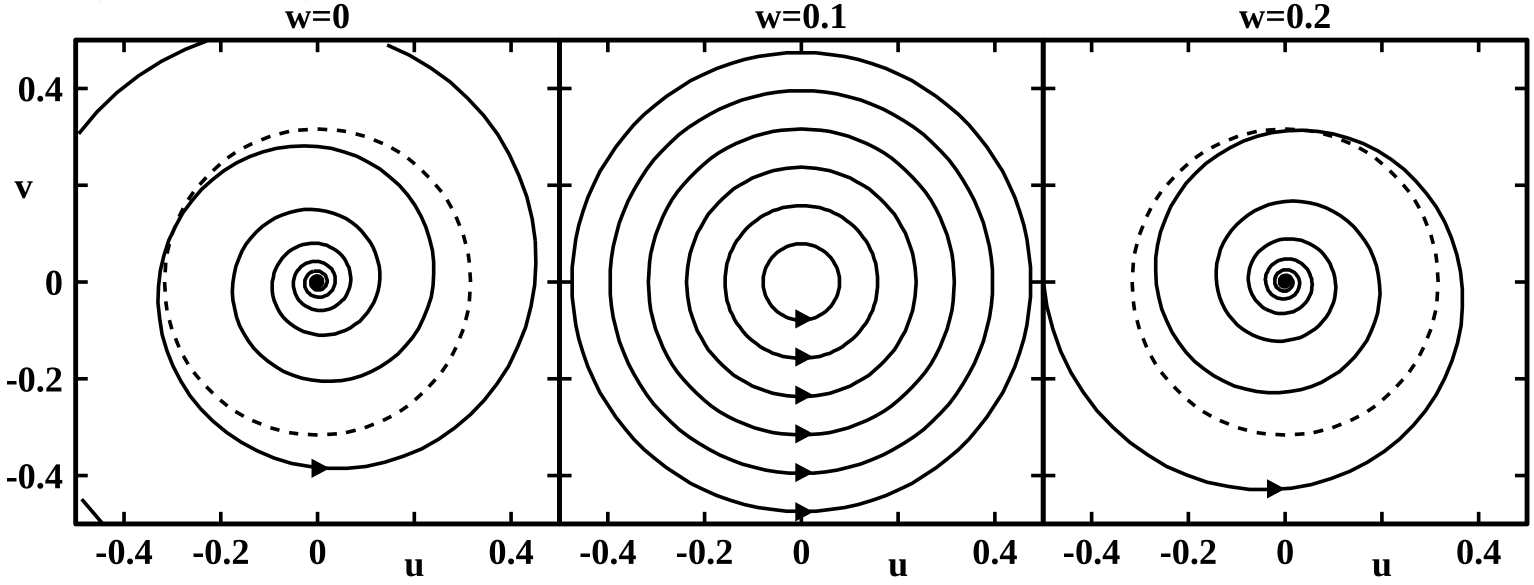
3-dim. SODE ($\mu^{\text{Ref}} = 0.1$)

$$\frac{du}{dt} = (\mu - w)\, u - v$$
$$\frac{dv}{dt} = (\mu - w)\, v + u$$
$$\frac{dw}{dt} = -w + (u^2 + v^2)$$

$\Longrightarrow$ **POD Galerkin system:**

$$\frac{da_1}{dt} = \left(\mu^{\text{GM}} - \mu^{\text{Ref}}\right) a_1 - a_2$$
$$\frac{da_2}{dt} = \left(\mu^{\text{GM}} - \mu^{\text{Ref}}\right) a_2 + a_1$$
$$\mu^{\text{Ref}} = 0.1$$

Galerkin solutions:



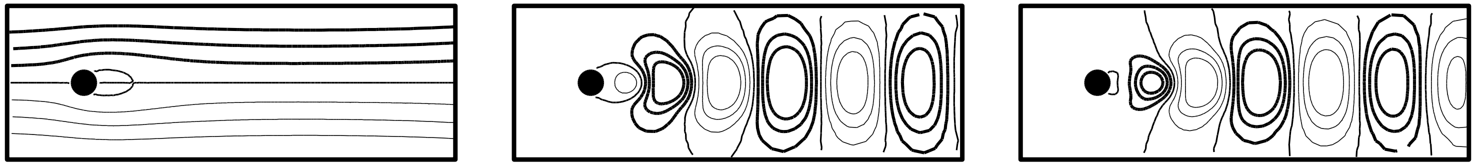
This 'toy' POD GM has all diseases of flow POD GM!

Transient dynamics of wake

≡ *Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —*

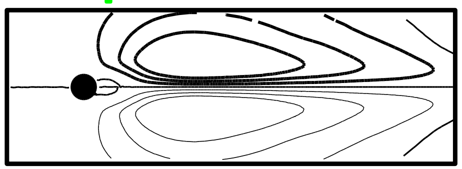
Operating condition II

on attractor



↑

transient on paraboloid



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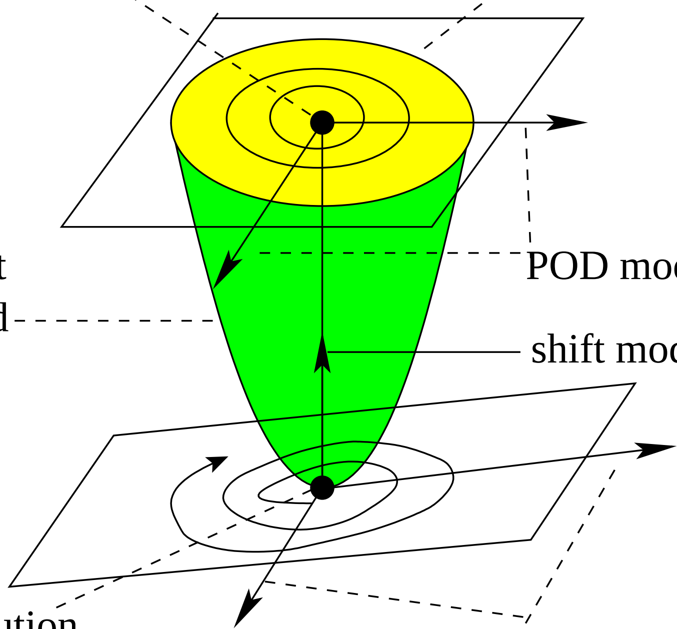
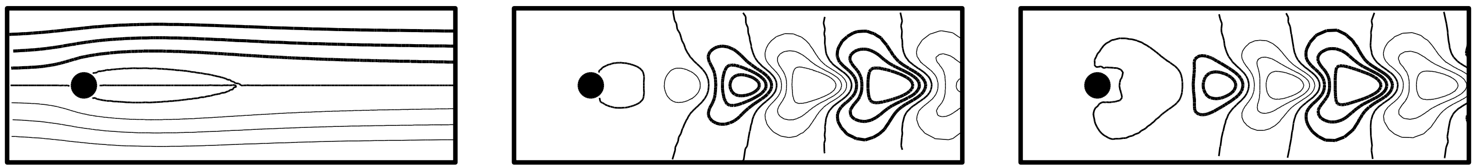


Operating condition I

near fixed point



≡ *Zielinska & Wesfreid (1995) PF*

Transient dynamics of GM with shift mode

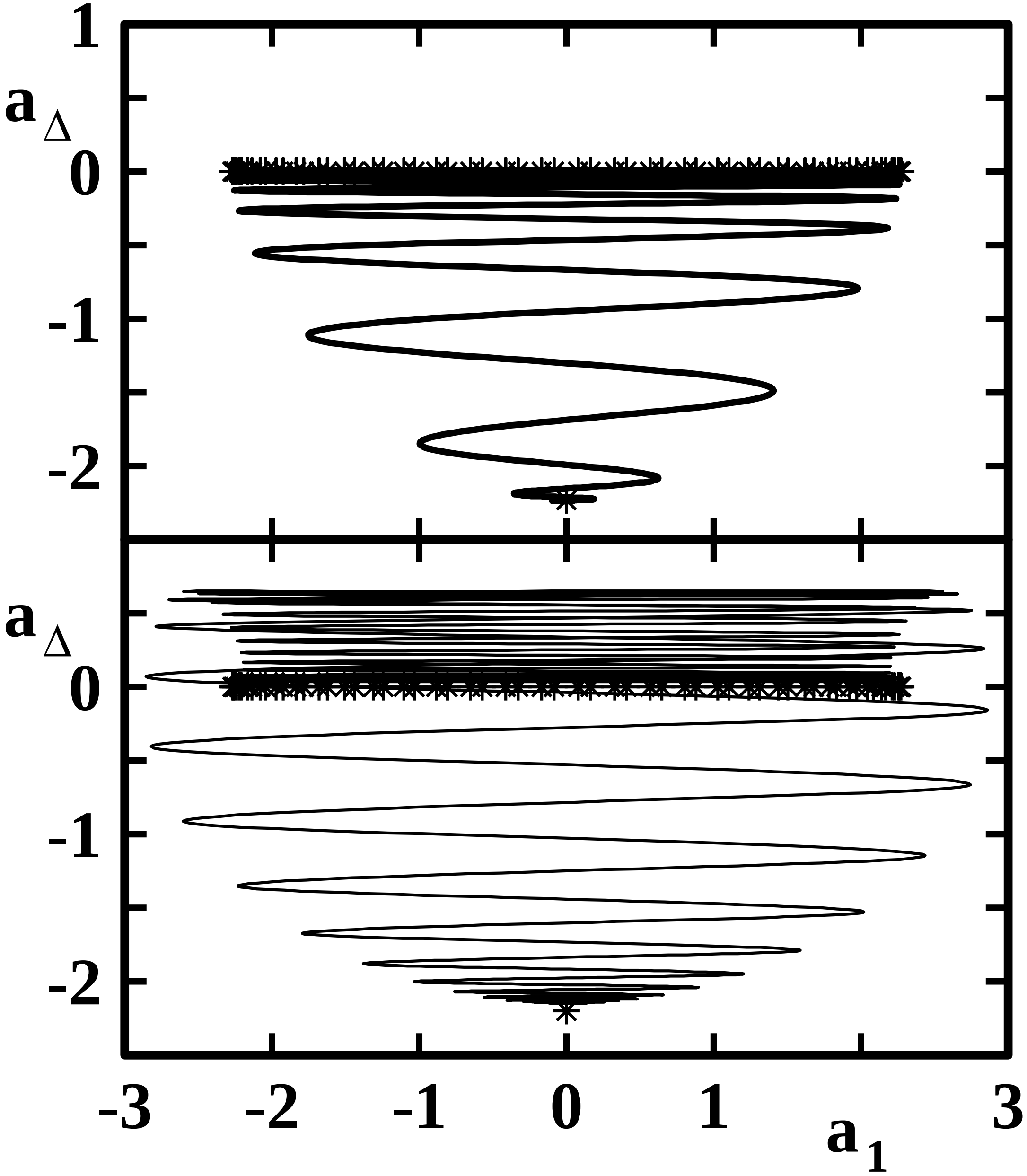
Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM

DNS . . . .

Galerkin model . . . .

8 POD modes

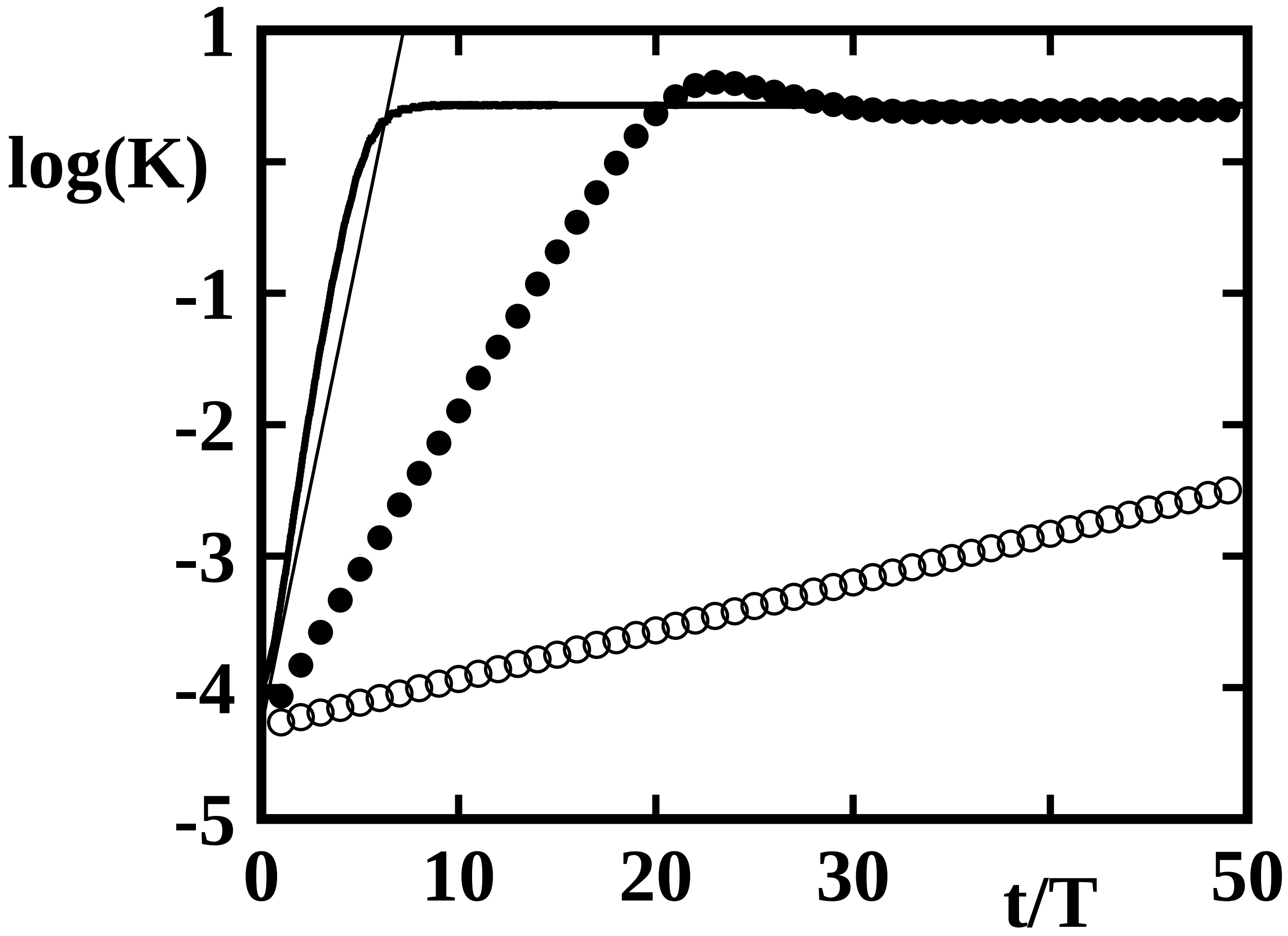
+1 shift mode



Shift mode in POD Galerkin models

— Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —

Transient dynamics



Amplitude selection mechanism

— ☰ Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —

(1) Stuart 1958

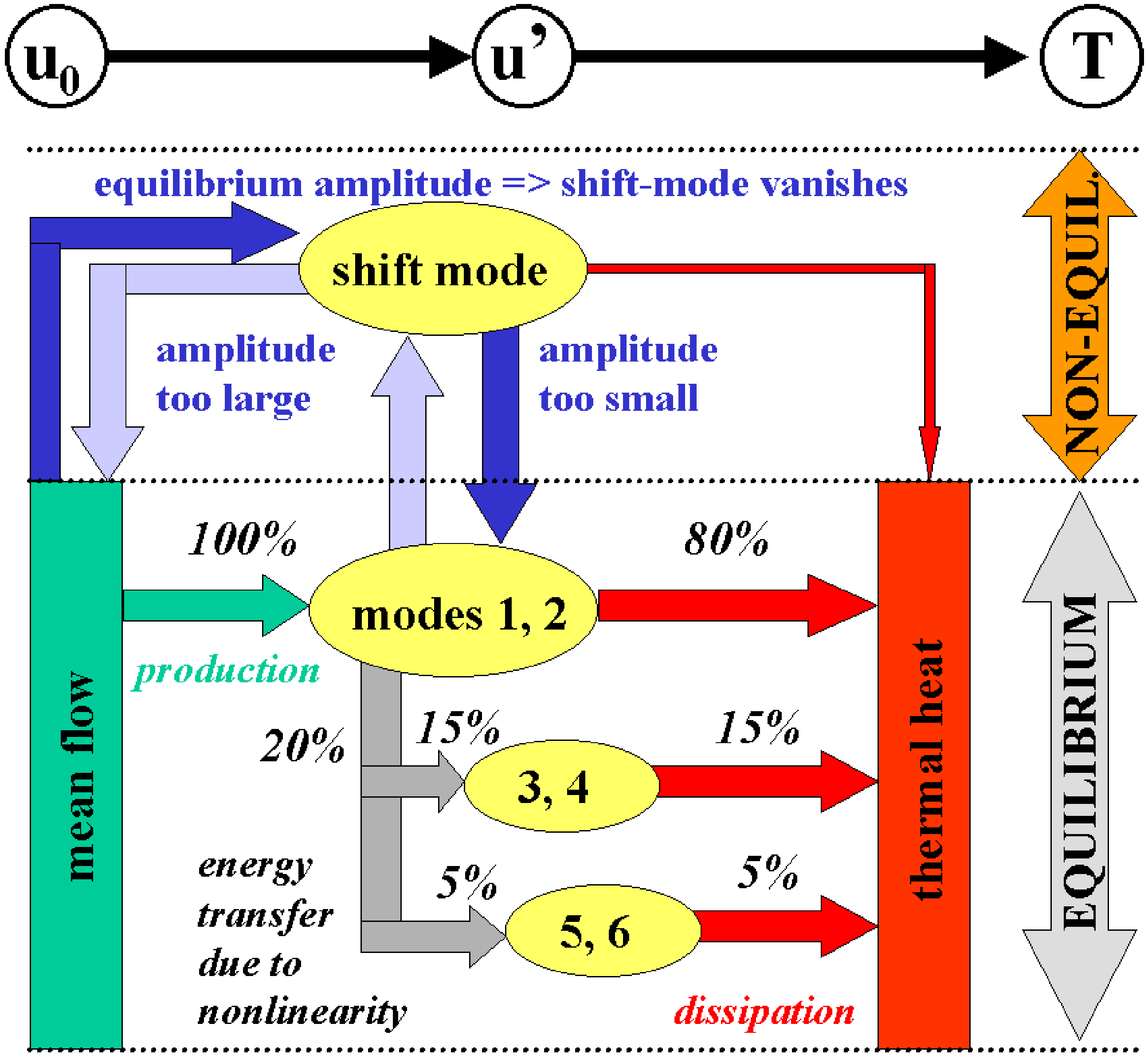
mean-field

theory

(2) ∼K41

energy flow

cascade



Re-dependency of GM with shift mode

— Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —

Strouhal frequency

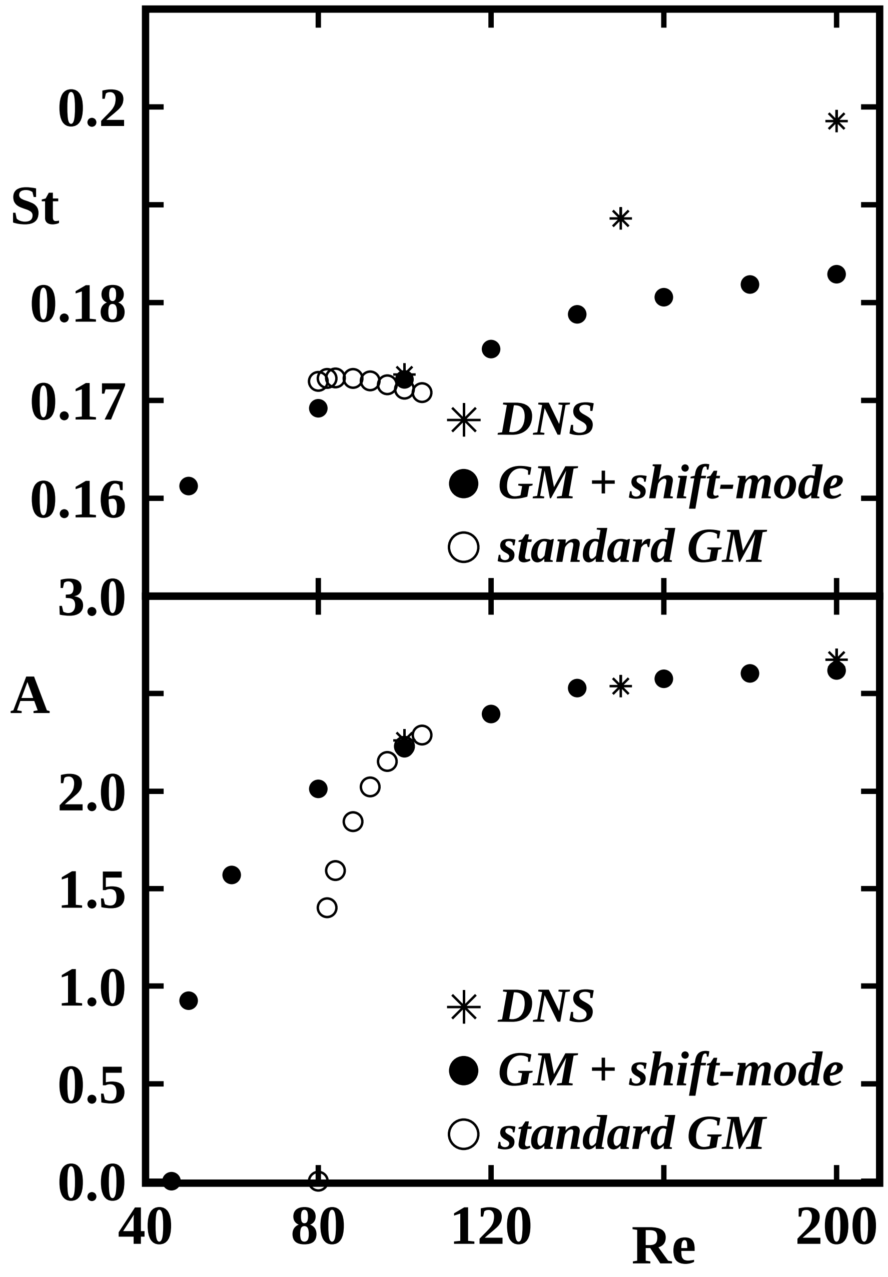
$St = Uf/D$ . . . .

- Trend $St(Re)$ well predicted with $\mathbf{u}_\Delta$.

Amplitude

$K = A^2/2$ . . . .

- Re_{crit} predicted with $\mathbf{u}_\Delta$.

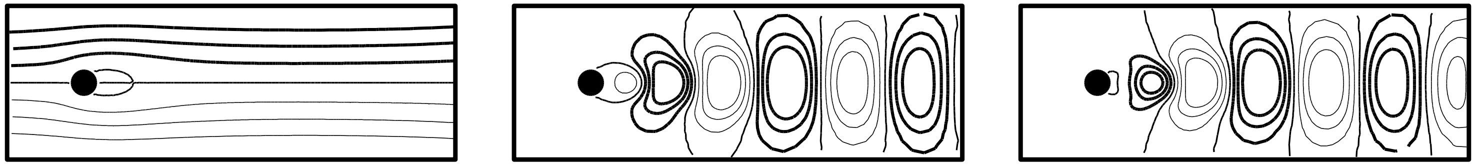


Transient dynamics of wake

≡ *Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —*

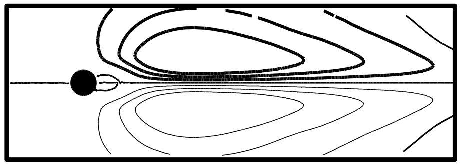
Operating condition II

on attractor



↑

transient on paraboloid



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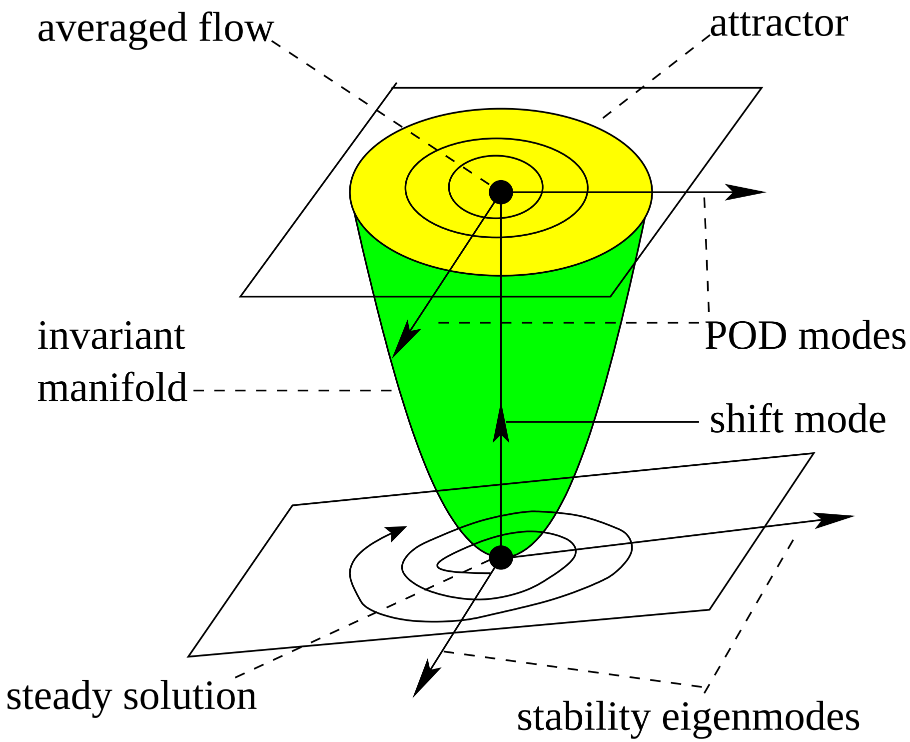
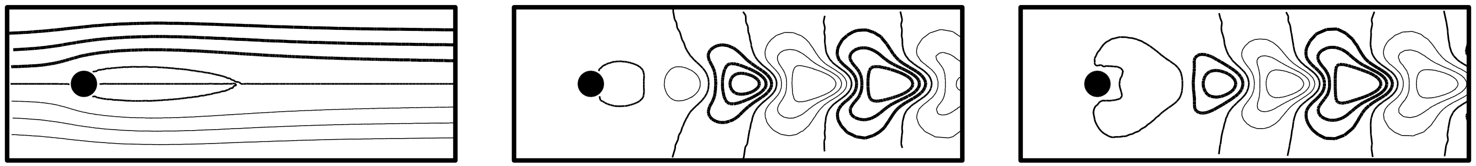


Operating condition I

near fixed point



≡ *Zielinska & Wesfreid (1995) PF*

Hybdrid GM from POD and mean-field modes

— Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM —

$$\mathbf{u} \rightarrow \partial_t \mathbf{u} = \nu \triangle \mathbf{u} - \nabla(\mathbf{u}\mathbf{u}) - \nabla p$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

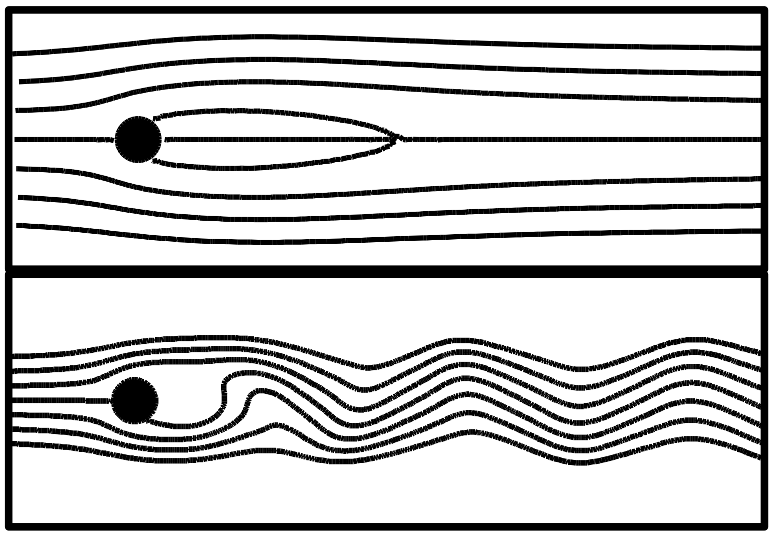
$$\mathbf{u} = \sum_{i=0}^{8} a_i \mathbf{u}_i \rightarrow \frac{da_i}{dt} = \nu \sum_{j=0}^{8+3} l_{ij} a_j + \sum_{j,k=0}^{8+3} \left(q^c_{ijk} + q^p_{ijk}\right) a_j\, a_k$$

$$+ a_\triangle \mathbf{u}_\triangle + \sum_{i=1}^{2} a_i^\star \mathbf{u}_i^\star \leftarrow + \text{ 1st stability eigenmode}$$

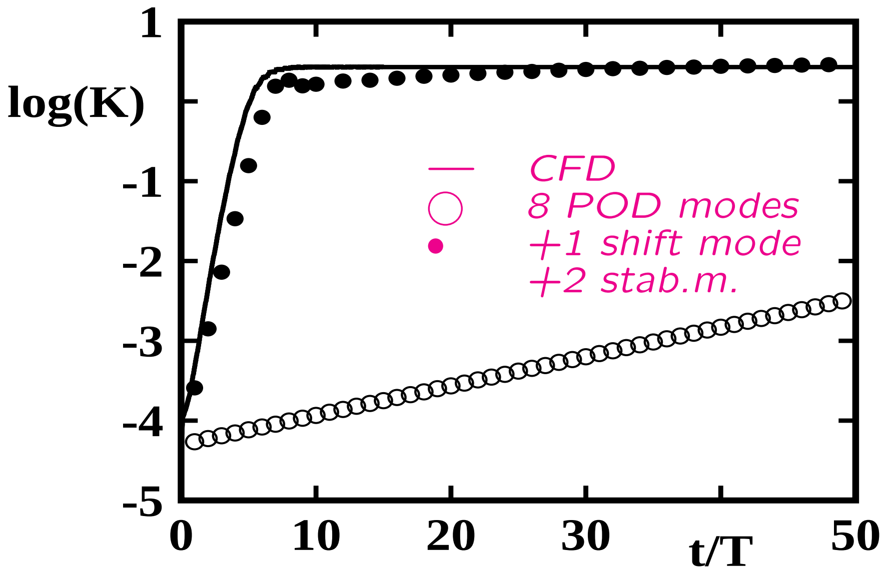
Kinematics

$t = 0 \rightarrow$

$t \geq 7T \rightarrow$



Transient dynamics



■ **Hybrid model resolves natural transients.**

Continuous mode interpolation

— *Morzyński, Stankiewicz, Noack, King, Thiele & Tadmor (2006) AFC* —

---

mode 1   mode 2

$\mathbf{u}_{1,2}^{\kappa=1}$

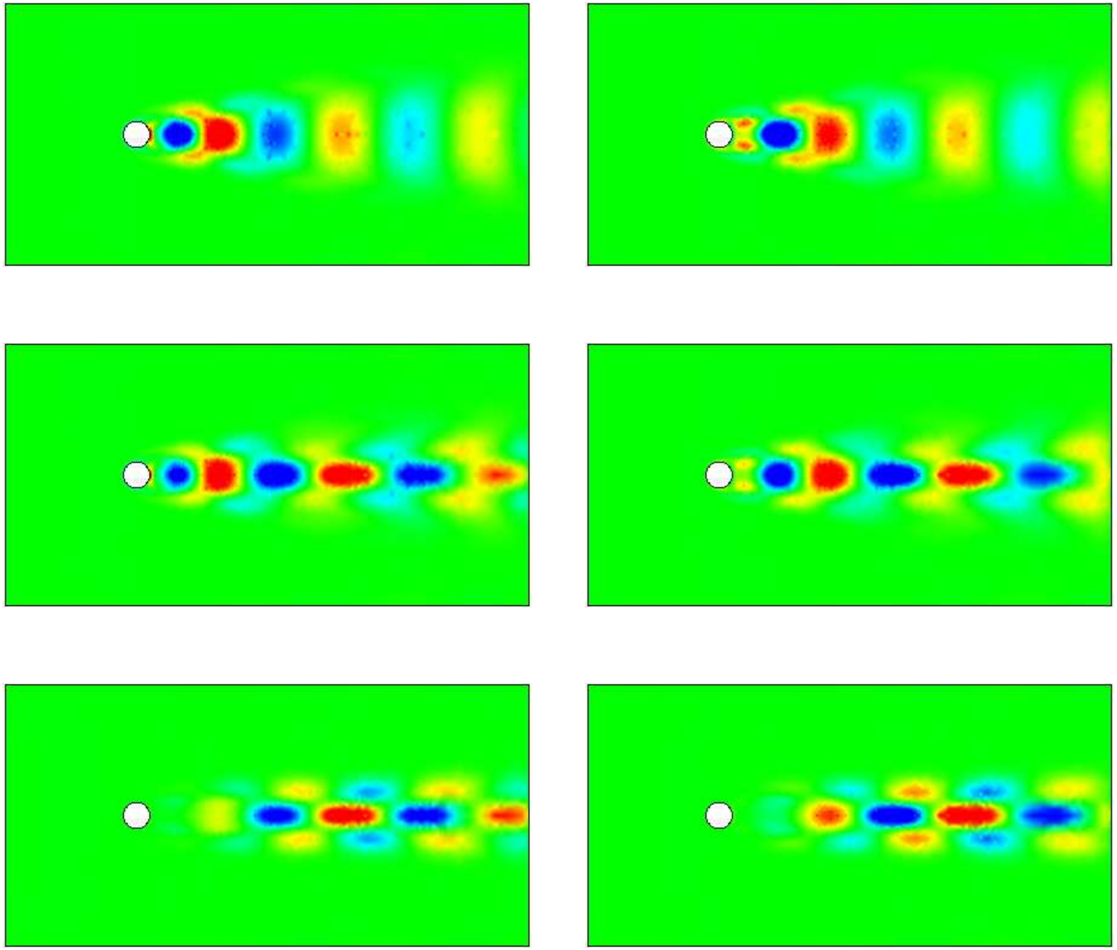
first POD modes

$\mathbf{u}_{1,2}^{\kappa=0.5}$

interpolated modes

$\mathbf{u}_{1,2}^{\kappa=0}$

stability eigenmode



- **Mode interpolation resolves intermediate states.**

Generalized mean-field model

— Morzyński, Stankiewicz, Noack, Thiele & Tadmor (2006) AIAA —

3-dim. Galerkin approx.:

$$\mathbf{u} = \mathbf{u}_B + \mathbf{u}'$$

$$\mathbf{u}_B = \mathbf{u}_s + a_\Delta \mathbf{u}_\Delta$$

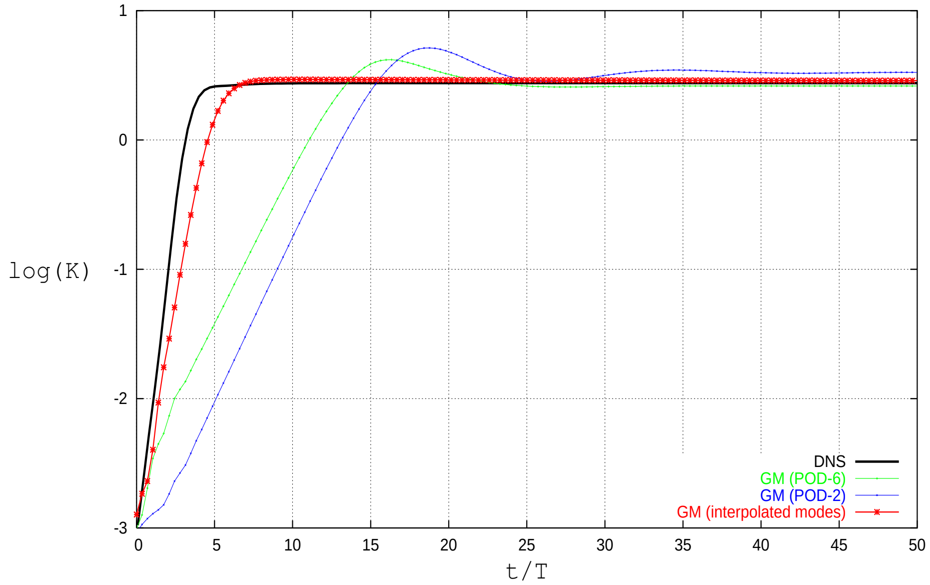
$$\mathbf{u}' = a_1^\kappa \mathbf{u}_1^\kappa + a_2^\kappa \mathbf{u}_2^\kappa$$

Galerkin system

$$da_1^\kappa/dt = \sigma a_1^\kappa - \omega a_2^\kappa$$

$$da_2^\kappa/dt = \sigma a_2^\kappa + \omega a_1^\kappa$$

Fluctuation energy for transient

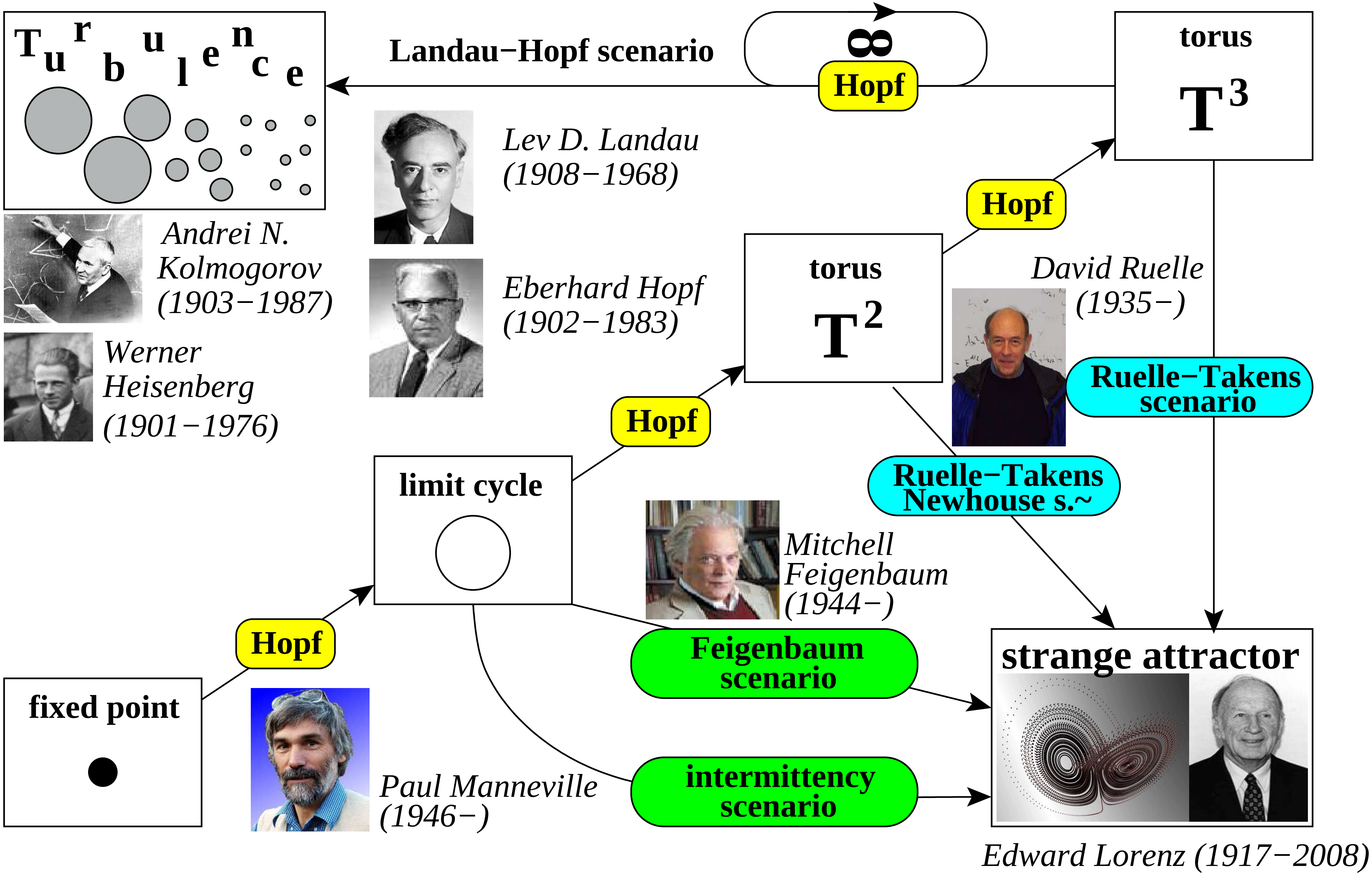


$$da_\Delta/dt = \sigma_D a_\Delta + c\left((a_1^\kappa)^2 + (a_2^\kappa)^2\right)$$

$$\sigma = \sigma_1 - \beta a_\Delta, \ \omega = \omega_1 + \gamma a_\Delta, \ \kappa = a_\Delta / a_\Delta^\infty$$

- **Generalized 3-dim. model $\sim$ 10% error.**

Dream #1: Instabilities $\mapsto$ turbulence



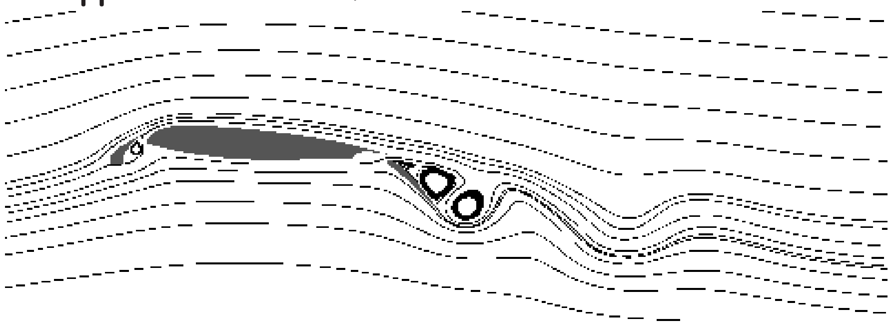
High-lift configuration

≡ *Luchtenburg, Günther, Noack, King & Tadmor (2008) JFM preprint*

Ask  **!**

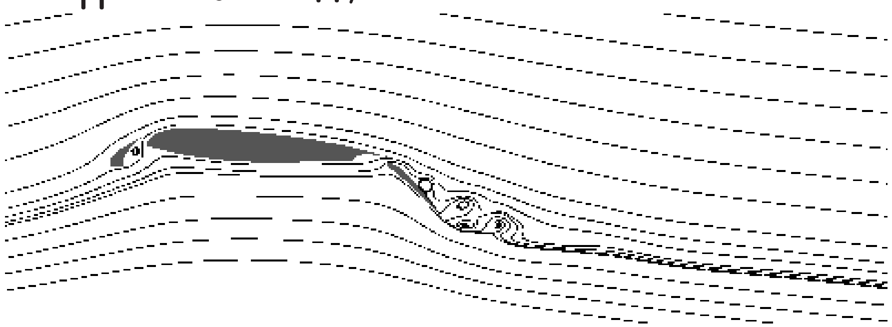
URANS $\mapsto$ natural flow

$$St^n_{\mathrm{fl}} = f^n c_{\mathrm{fl}}/U_\infty = 0.32$$



actuated flow

$$St^a_{\mathrm{fl}} = f^a c_{\mathrm{fl}}/U_\infty = 0.6$$

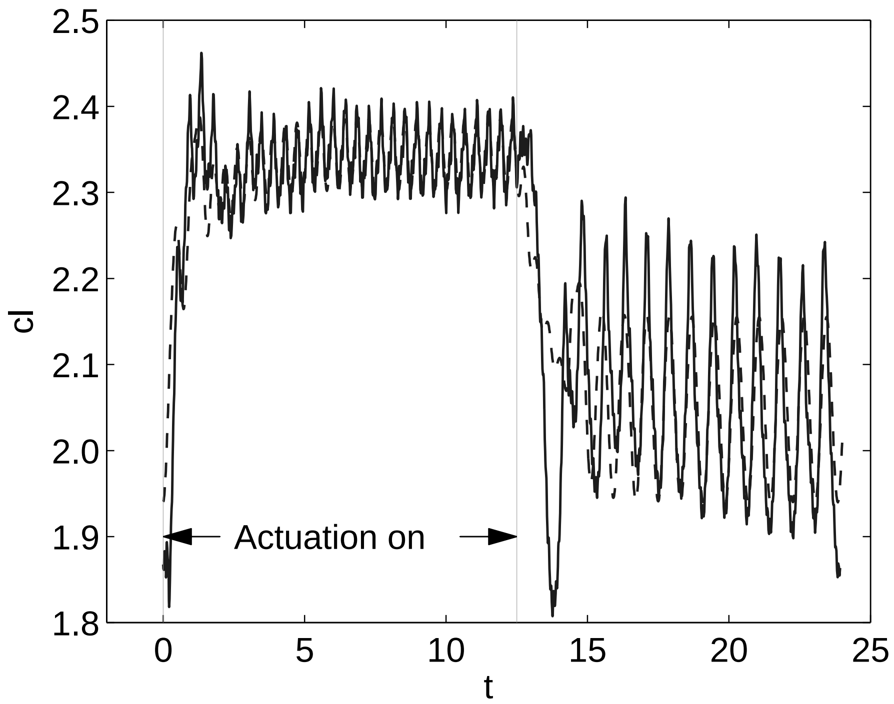


Galerkin model

$$\mathbf{u} = \mathbf{u}_0 + \sum_{i=1}^{5} a_i(t)\,\mathbf{u}_i(\mathbf{x})$$

$$d\mathbf{a}/dt = f(\mathbf{a},\mathbf{b}), \quad \mathbf{b}: \text{control}$$

$$c_L = c_L(\mathbf{a}) \text{ lift coefficient}$$



Mean field model can explain ...

- **Transient dynamics**
- **Amplitude selection mechanism**

  (1) dynamics . . . **Landau equation** $dA/dt = \sigma A - \beta A^3$;

  (2) momentum . . . . . . **Reynolds equation;** $a_\triangle = cA^2$;

  (3) energetics . . . . $dK/dt = P_1 + a_\triangle P_2 - D$;

- **Malkus (1958) principle of marginal stability**

  for the mean flow

- **Structural instability** of POD models

  ... and the need for shift/non-equilibrium modes

- **Many more things:**

  local linearisation, laminar verus turbulent channel flow, Questions?

Comments?

Stefan Siegel's
coffee break!