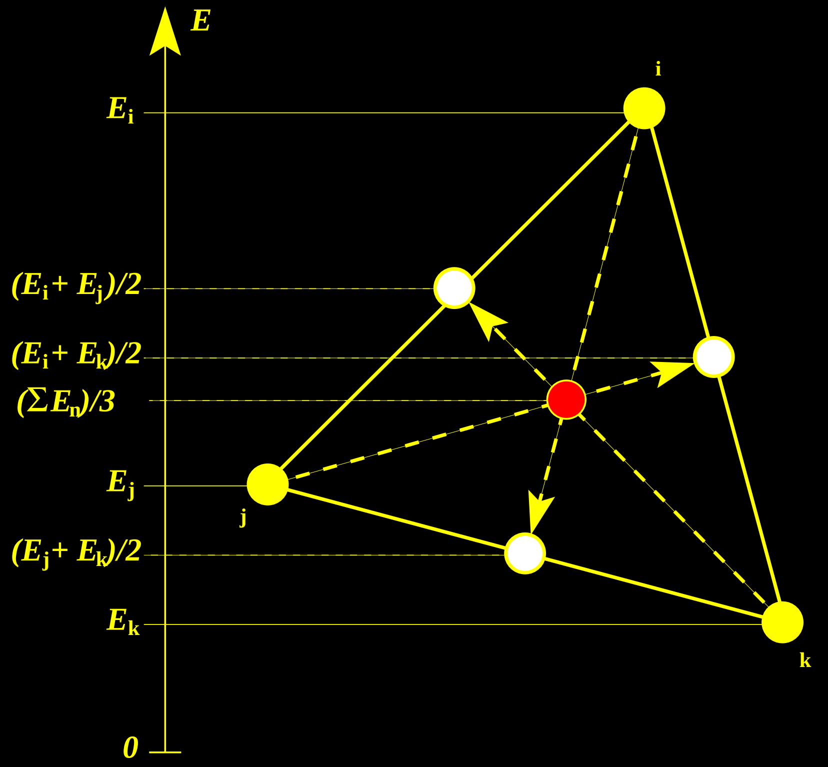
Low-dimensional modelling
— Towards attractor control II

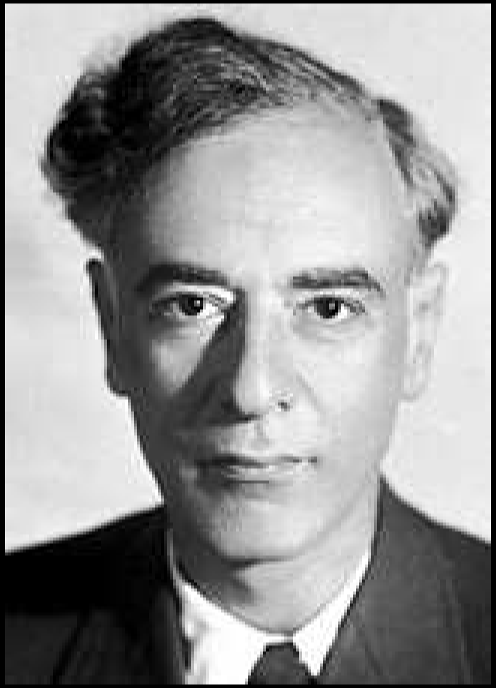


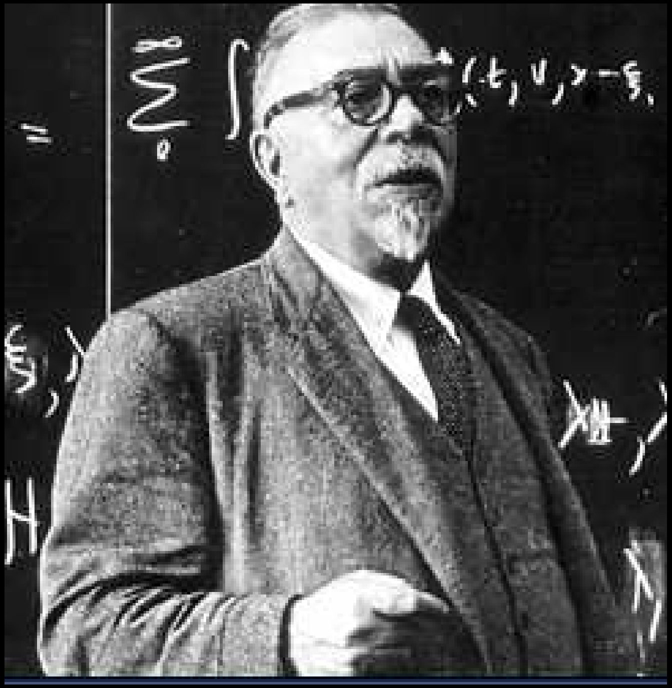
Bernd R. Noack & friends

Berlin Institute of Technology & *elsewhere*

CISM course "Reduced-Order Modelling for Flow Control", Udine, September 15–19, 2008

Low-dimensional modelling
— Towards attractor control





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CISM course "Reduced-Order Modelling for Flow Control", Udine, September 15–19, 2008

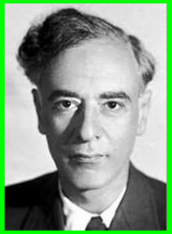
Overview

1. **Introduction**

   . . . . *physics & cybernetics dreams revisited*

2. **Mean-field modelling**

   . . . . *complete order / stability theory*

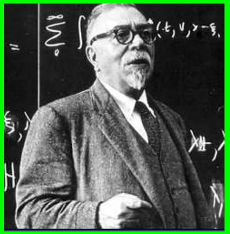


3. **Attractor modelling**

   . . . . *complete disorder / statistical physics*

4. **Attractor control**

   . . . . *Maxwellian and other deamons*



5. **Summary and outlook**

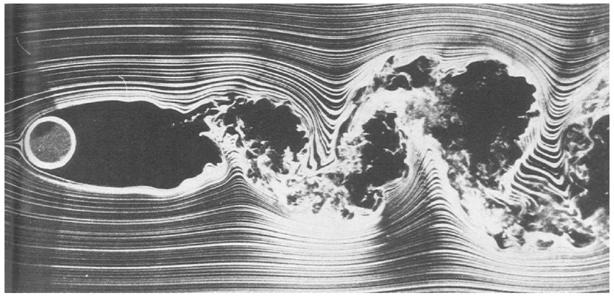
Dream #2: Statistical physics $\mapsto$ turbulence

Ludwig Boltzmann
(1840–1906)
Equivalent subsystems:
1877: Entropy

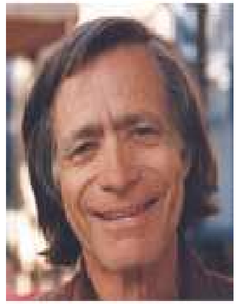
$S = k\, lnW$

Lars Onsager
(1903–1976)
Particle/vortex picture:
1949: point vortices in 2D flows
= thermodyn. degree of freedom

Hans W Liepmann (1914-)'s
WARNING:
Don't forget $\rightarrow$



Robert H Kraichnan
(1928–2008)
Wave/Galerkin picture:
1955: Fourier modes
= thermodyn. degrees of freedom
(absolute equilibrium ensemble)



How to partition the flow in equivalent subsystems (atoms)
(= thermodynamic degrees of freedom)???

'Traditional' Galerkin method

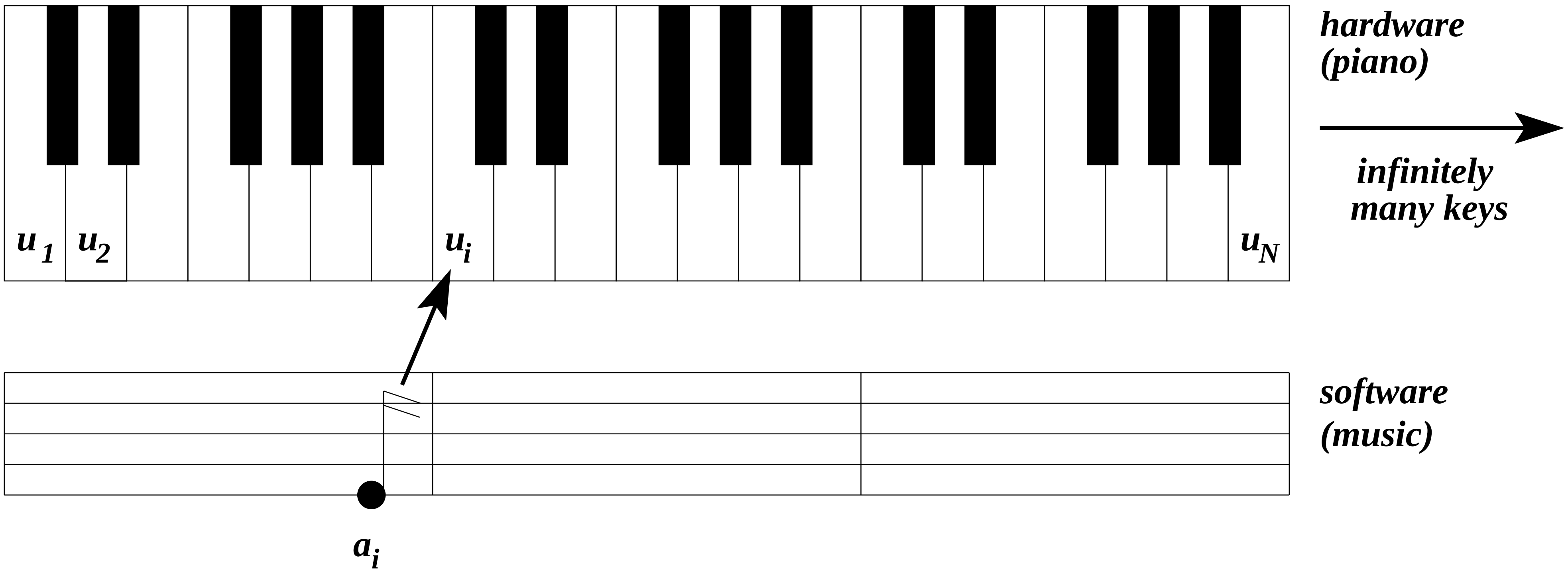
— *Fletcher 1984 Computational Galerkin Methods, Springer* —

Galerkin method

$$\mathbf{u}(\mathbf{x},\mathbf{t}) \quad \rightarrow \quad \partial_t \mathbf{u} = \frac{1}{Re}\triangle \mathbf{u} - \nabla(\mathbf{u}\mathbf{u}) - \nabla p$$

$$\downarrow \qquad\qquad \downarrow$$

$$\mathbf{u}^{[\mathrm{N}]} = \sum_{i=0}^{N} a_i(t)\mathbf{u}_i(\mathbf{x}) \quad \rightarrow \quad \frac{da_i}{dt} = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$



Finite-time thermodynamics formalism

— *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

dynamical system

constant

$\frac{da_i}{dt} = c_i + \ldots$

linear term

$+ \sum_j l_{ij} a_j + \ldots$

energy preserving

quadr. term

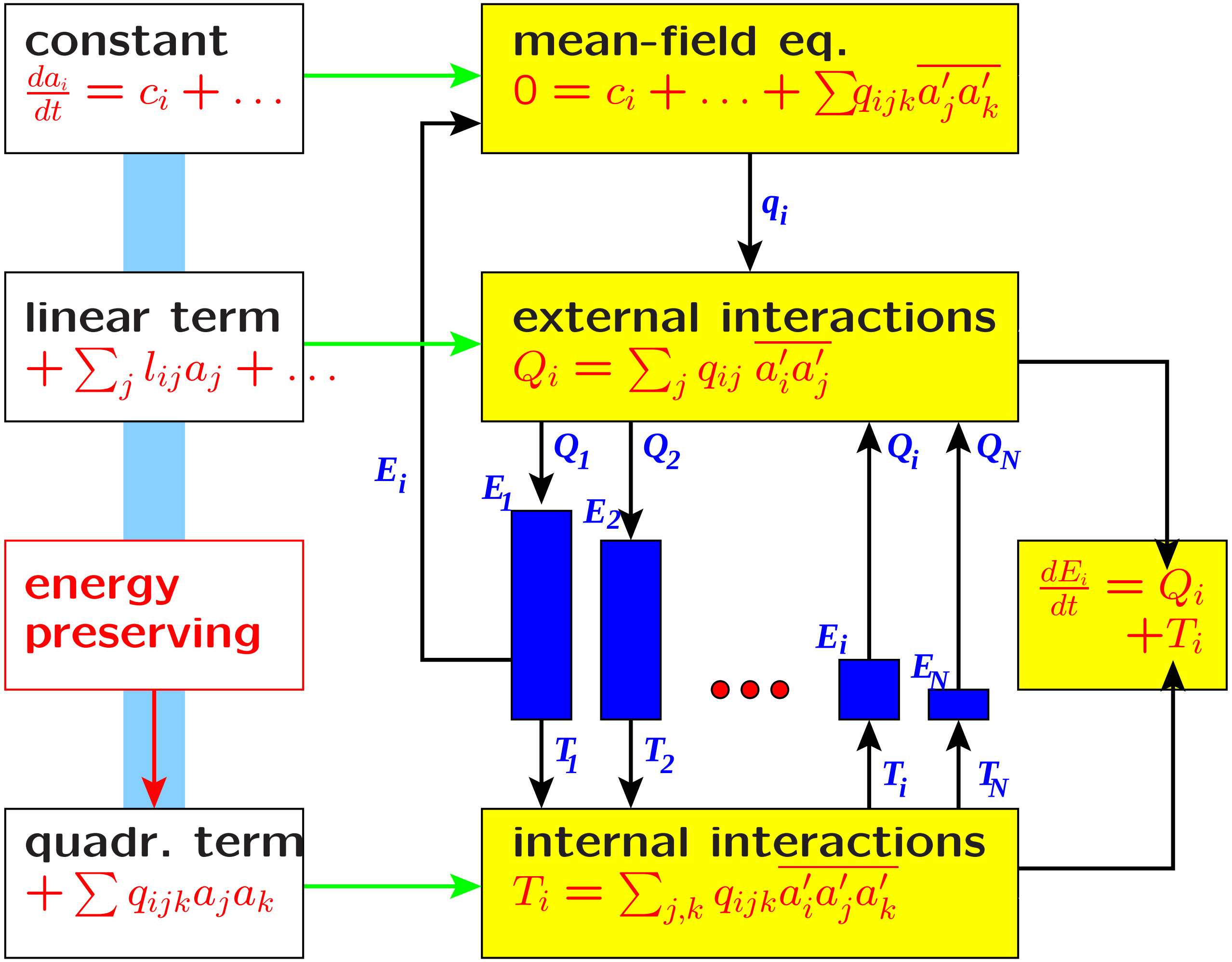
$+ \sum q_{ijk} a_j a_k$

Finite-time thermodynamics formalism

— ≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

dynamical system **averaged equations**

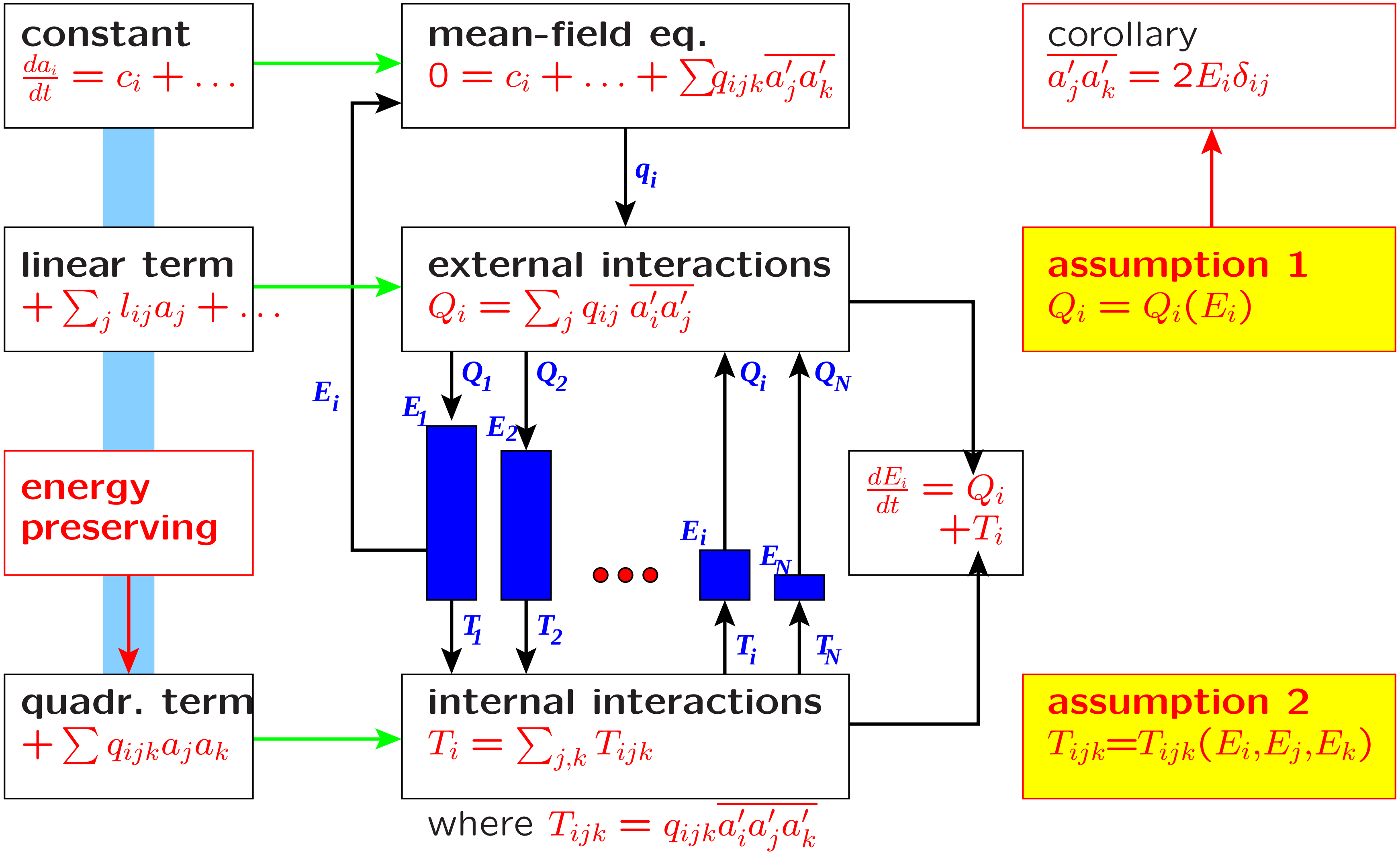
$a_i = \overline{a_i} + a'_i,\ E_i = \overline{(a'_i)^2}/2$



Finite-time thermodynamics formalism

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

dynamical system | **averaged equations** $a_i = \overline{a_i} + a_i'$, $E_i = \overline{(a_i')^2}/2$ | **closure assumptions**

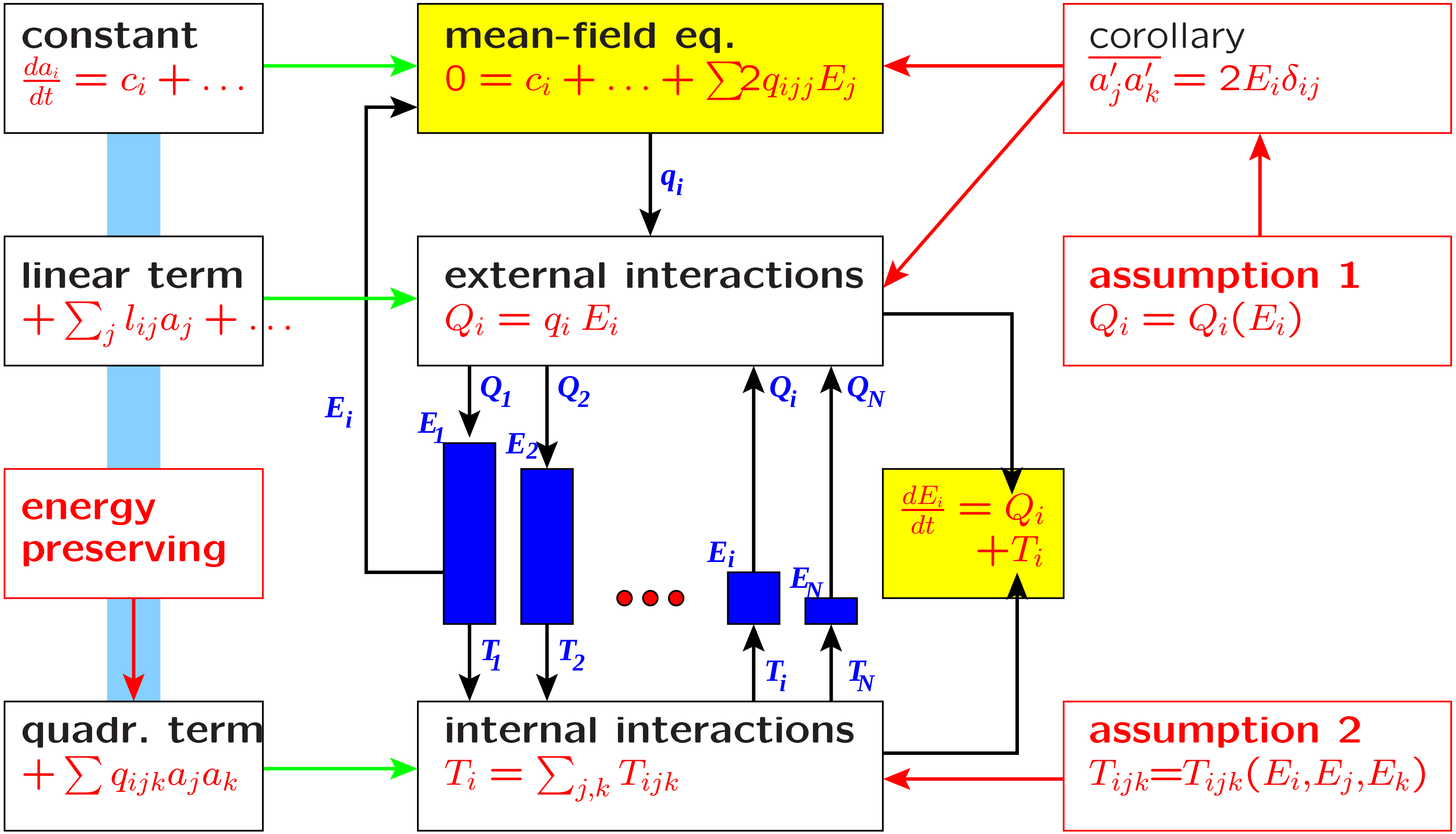


Finite-time thermodynamics formalism

— Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

dynamical system | **FTT equations** | **closure assumptions**

$a_i = \overline{a_i} + a_i', \; E_i = \overline{(a_i')^2}/2$



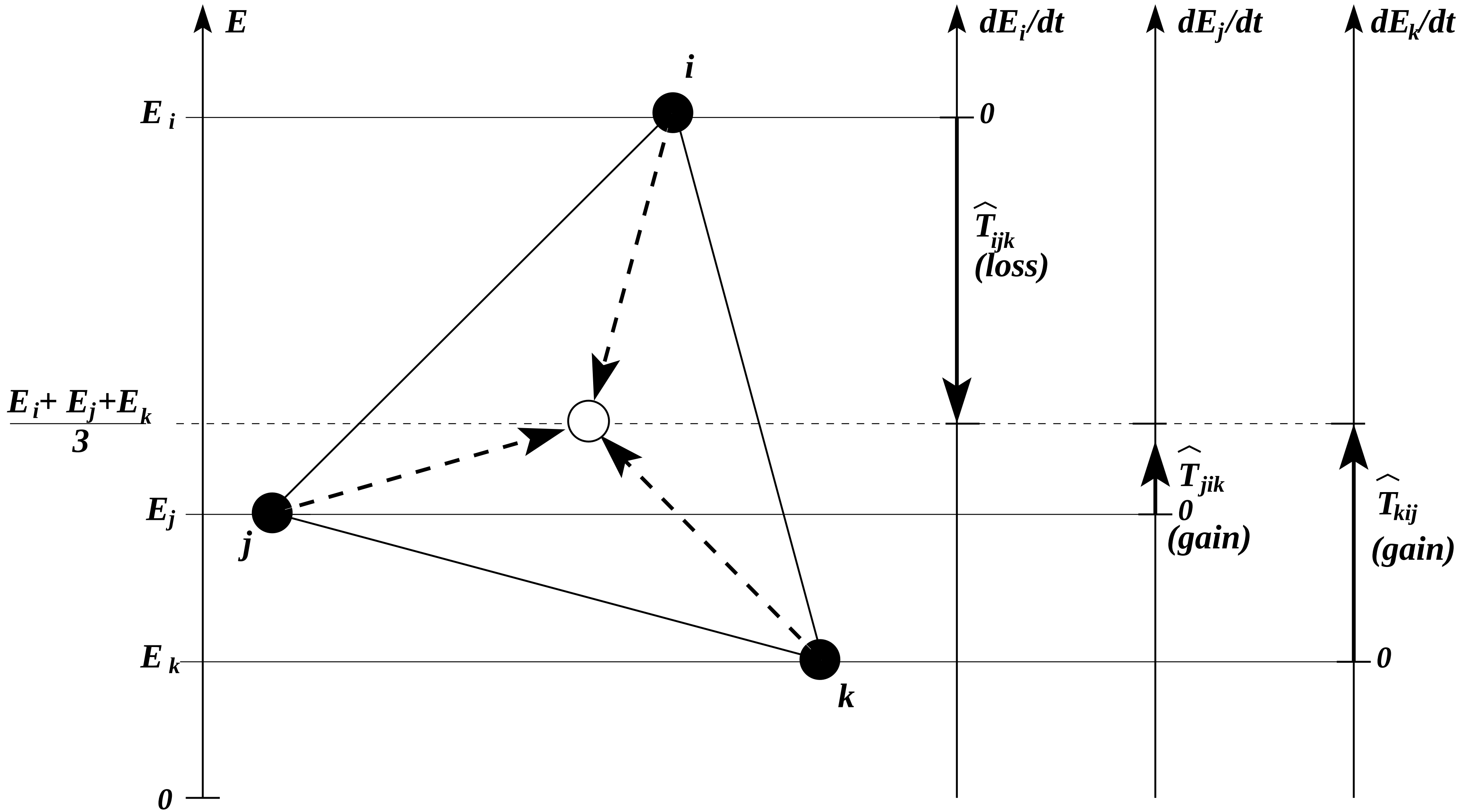
where $T_{ijk} = \alpha \, \chi_{ijk} \times$

$$\sqrt{E_i E_j E_k} \left[1 - \frac{3E_i}{E_i + E_j + E_k} \right]$$

Fick's law of triadic interactions

— ☰ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

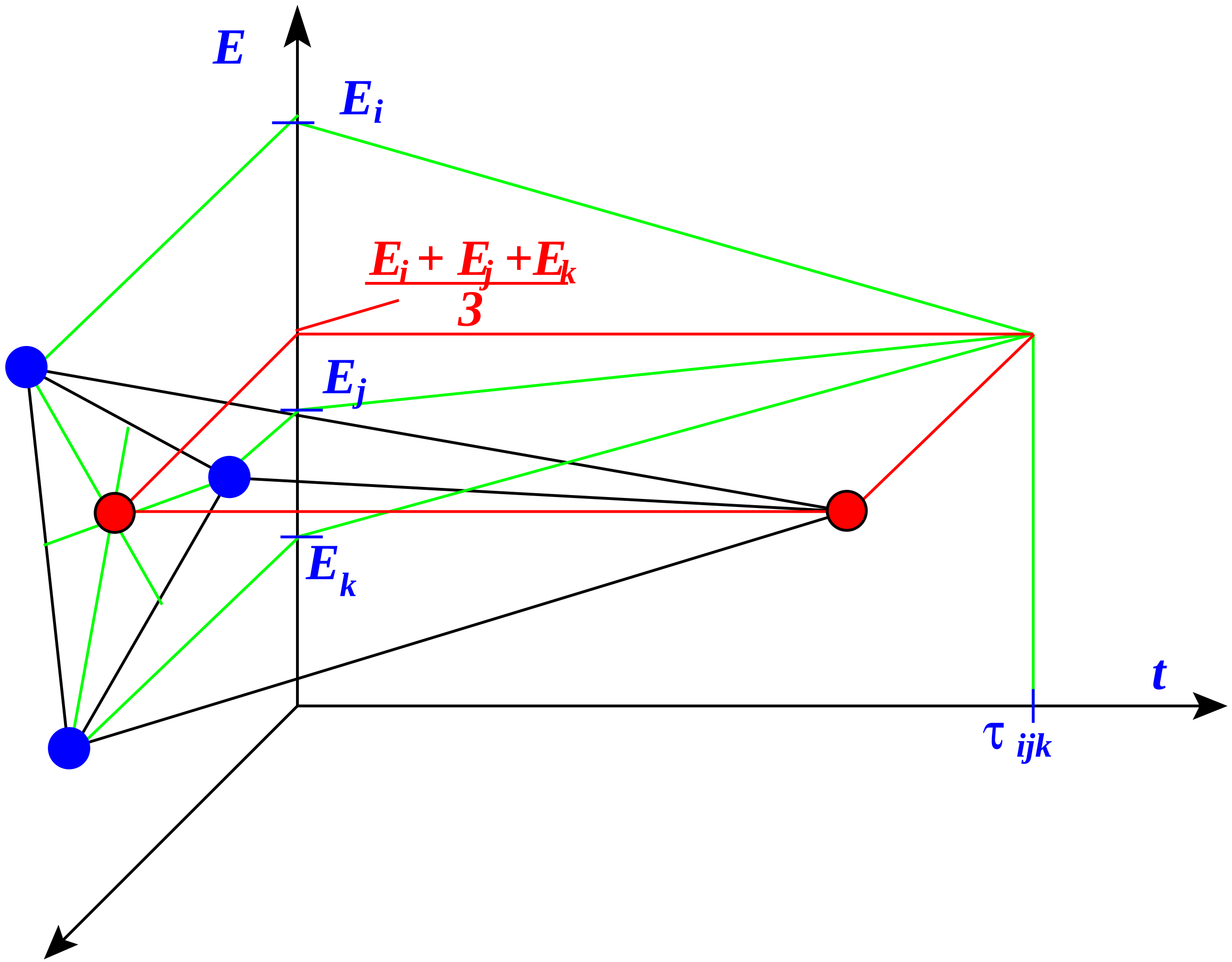
$$T_{ijk} = \sigma_{ijk} \left[1 - \frac{3E_i}{E_i + E_j + E_k} \right], \quad \text{where} \quad \sigma_{ijk} = \alpha \, \chi_{ijk} \sqrt{E_i E_j E_k}$$



Fick's law of triadic interactions

— ≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

$$T_{ijk} = \sigma_{ijk} \left[1 - \frac{3E_i}{E_i + E_j + E_k} \right], \quad \text{where} \quad \sigma_{ijk} = \frac{1}{\tau_{ijk}} = \alpha\, \chi_{ijk} \sqrt{E_i E_j E_k}$$



Fick's law for triadic interactions

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

Ansatz

$$T_{ijk} = T_{ijk}(E_i, E_j, E_k)$$

Properties from analysis of $T_{ijk} = q_{ijk}\, \overline{a_i a_j a_k}$

(1) Homogeneity . . . . $T_{ijk}(\lambda E_i, \lambda E_j, \lambda E_k) = \lambda^{3/2}\, T_{ijk}(E_i, E_j, E_k)$

(2) Zeros . . . . $T_{ijk}(E_i, E_j, 0) = T_{ijk}(E_i, 0, E_k) = T_{ijk}(0, E_j, E_k) = 0$

(3) Symmetry . . . . $T_{ijk} = T_{ikj}$

(4) Monotonicity . . . . $E_i < \min\{E_j, E_k\} \quad \Rightarrow T_{ijk}(E_i, E_j, E_k) < 0$

(5) Energy preservation . . . . $T_{ijk} + T_{ikj} + T_{jik} + T_{jki} + T_{kij} + T_{kji} = 0$

(6) Realizability (strictly: $|T_{ijk}| \le |q_{ijk}|\, |a_i|\text{max}\, |a_j|\text{max}\, |a_k|\text{max}$)

$$|T_{ijk}| \lesssim |q_{ijk}|\, \sqrt{E_i\, E_j\, E_k}$$

Solution

$$T_{ijk} = \alpha \chi_{ijk}\, \sqrt{E_i E_j E_k}\, \left[1 - \frac{3E_i}{E_i + E_j + E_k}\right]$$

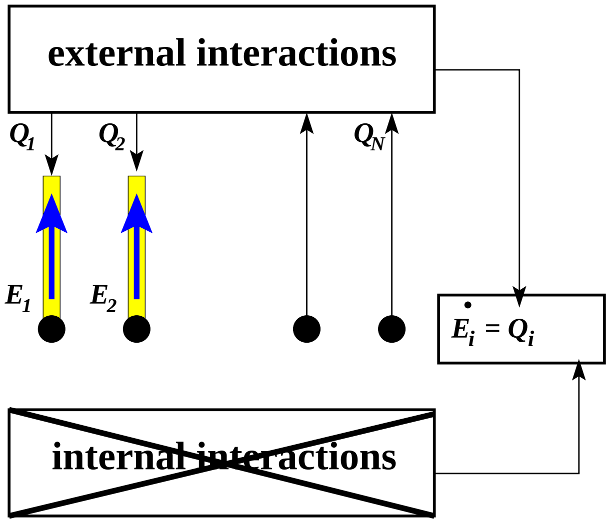
with the totally symmetric triadic structure function

$\chi_{ijk} := \frac{1}{6}\Big(|q_{ijk}| + |q_{ikj}| + |q_{jik}| + |q_{jki}| + |q_{kij}| + |q_{kji}|\Big)$ and α determined from energy flow consistency between donor and recipient modes.

Finite-time thermodynamics — limits

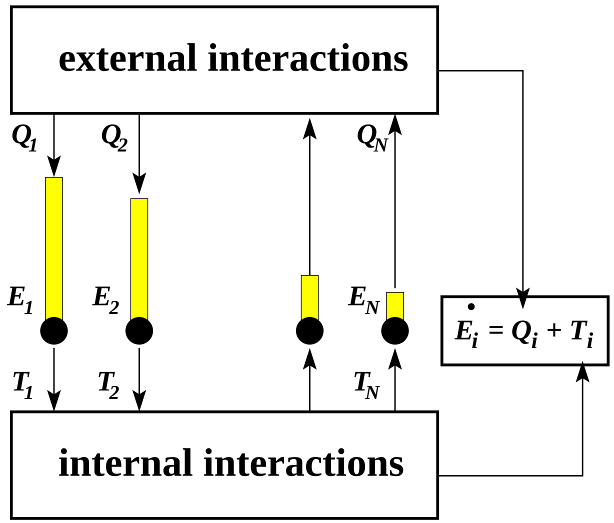
— Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

Linear dynamics



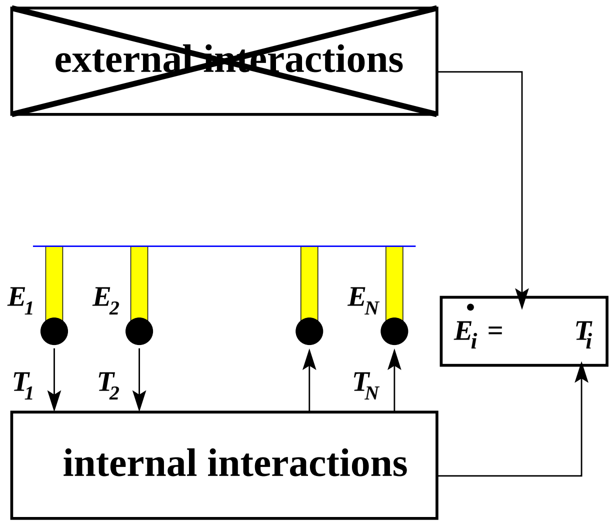
Time-scale
for E-growth
$\ll$
time-scale
for E-redistribution

Partial thermal equilibrium



Time-scale
for E-growth
$\sim$
time-scale
for E-redistribution

thermal equilibrium



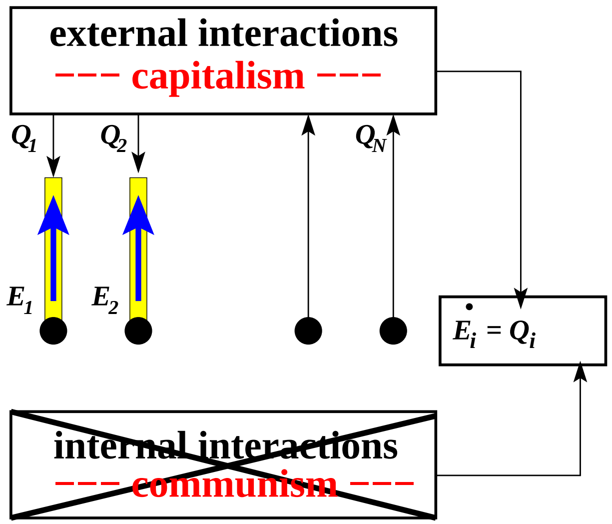
Time-scale
for E-growth
$\gg$
time-scale
for E-redistribution

Andresen, Salamon & Berry 1977 JCP: Thermodynamics in finite time...

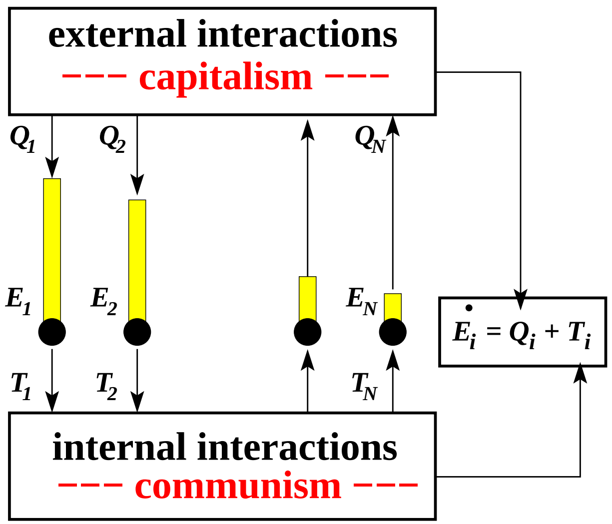
Finite-time thermodynamics — limits

— Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

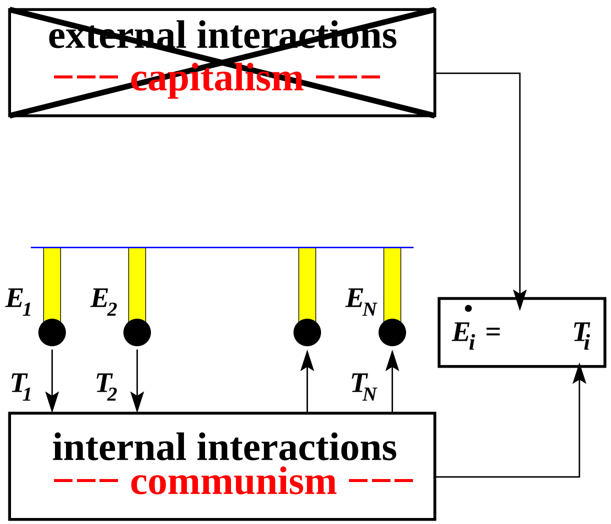
Linear dynamics



Partial TE



TE



— Economics analogy —

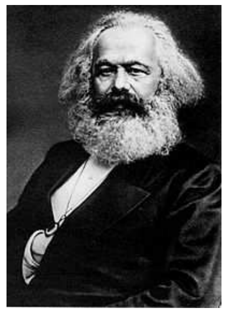
Neo liberalism

Warren Buffett (1930–)

Social market e.∼

Luwdig Erhard (1897–1977)

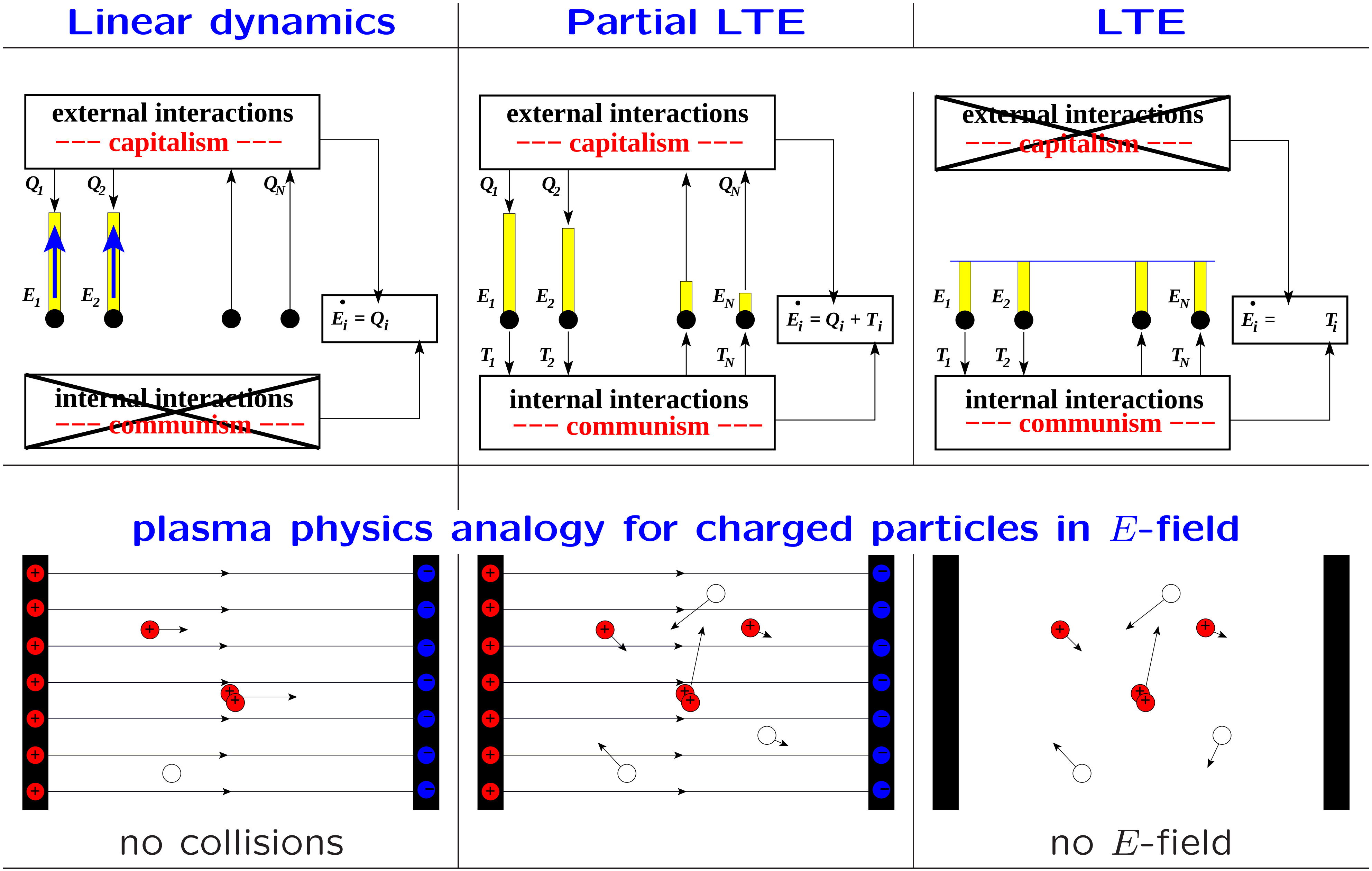
Communism



Karl Marx (1818–1883)

FTT model — extremal limits

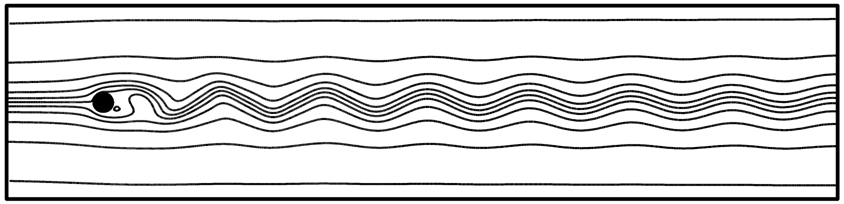
— ≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —



Periodic cylinder wake ($Re = 100$)

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

2D flow around circular cylinder (DNS)

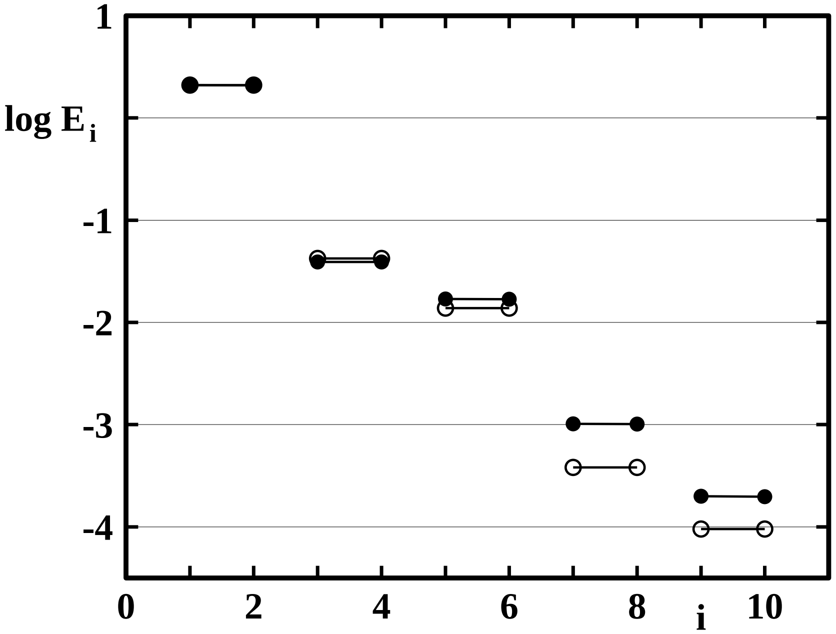


10-dim. Galerkin model

$$\mathbf{u} = \sum_{i=0}^{N} a_i \mathbf{u}_i \quad \text{(POD modes)}$$

$$\dot{a}_i = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$

Energy distribution (computed and FTT predicted)



•: DNS; ∘: FTT

Good agreement between DNS and FTT prediction!

Burgers' equation

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

Boundary value problem

$$\partial_t u + (U + u)\,\partial_x u = g(x,t) + \nu\,\partial^2_{xx} u$$

$U = 1$, $\nu = 1/100$, energy source $g(x,t) = \sigma\ (a_1\,\Theta_1 + a_2\,\Theta_2)$, $\sigma = 1/50$.

BC: $u(x + 2\pi, t) = u(x,t)$

Galerkin approximation (here: $N = 10$, 1st to 5th harmonics)

$$u(x,t) = a_0(t)\Theta_0(x) + a_1(t)\,\Theta_1(x) + \ldots a_N\,\Theta_N(x)$$

$\Theta_0 = \frac{1}{\sqrt{\pi}}$, $\Theta_1 = \frac{1}{\sqrt{2\pi}}\sin x$, $\Theta_2 = \frac{1}{\sqrt{2\pi}}\cos x$, $\Theta_3 = \frac{1}{\sqrt{2\pi}}\sin 2x$, ...

Galerkin system: $\dot{a}_0 = 0$

$$\dot{a}_i = \sum_{j=1}^{N} l_{ij}\,a_j + \sum_{j,k=1}^{N} q_{ijk}\,a_j\,a_k$$

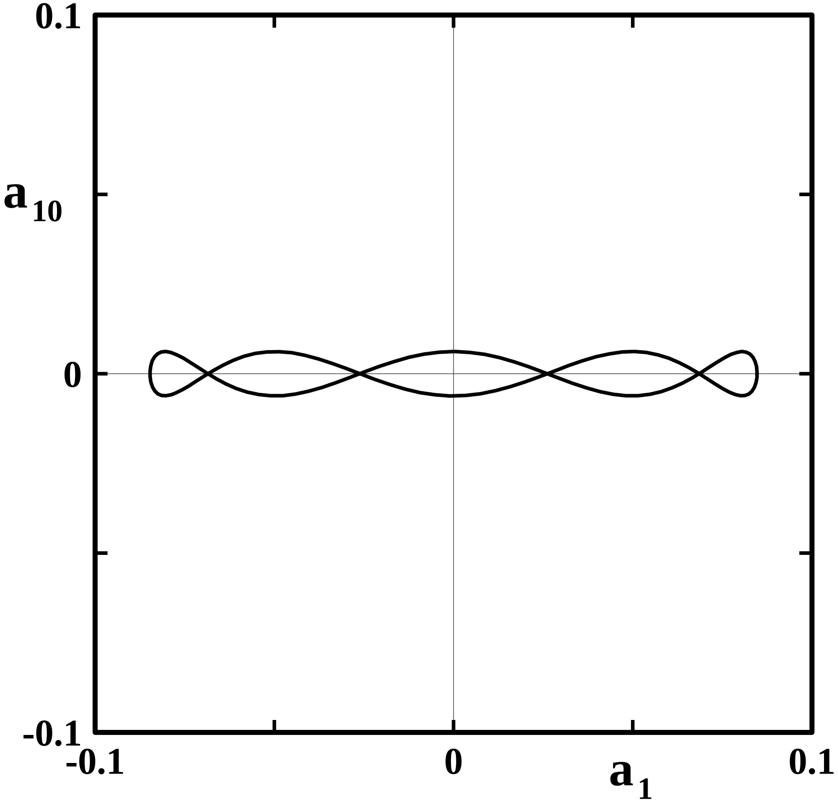
nonlinearly coupled oscillators ($i = 1,2$: self-excited, $i \geq 3$: damped)

Burgers' equation II

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

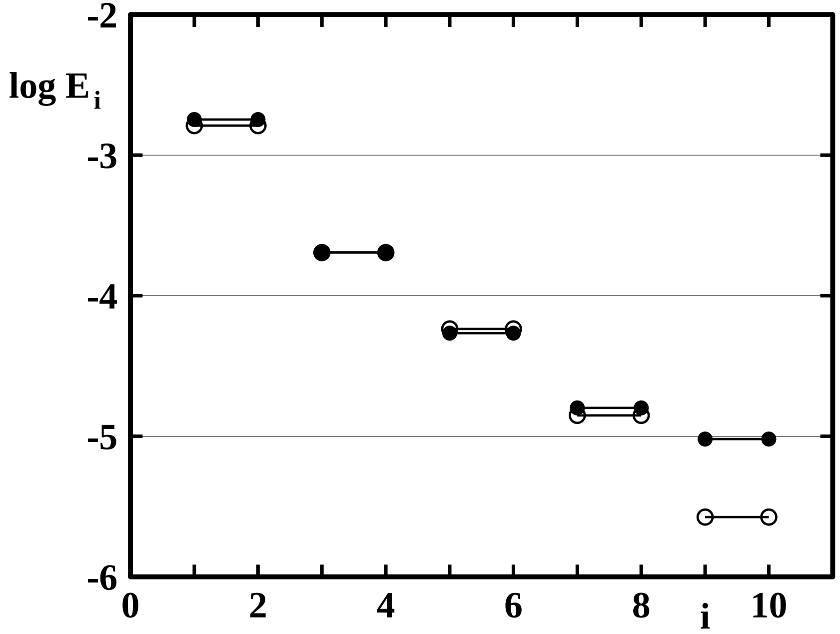
Travelling wave solution with energy source and diffusion term

phase portrait



$U = 1$, $\sigma = 1/50$, $\nu = 1/100$

energy distribution



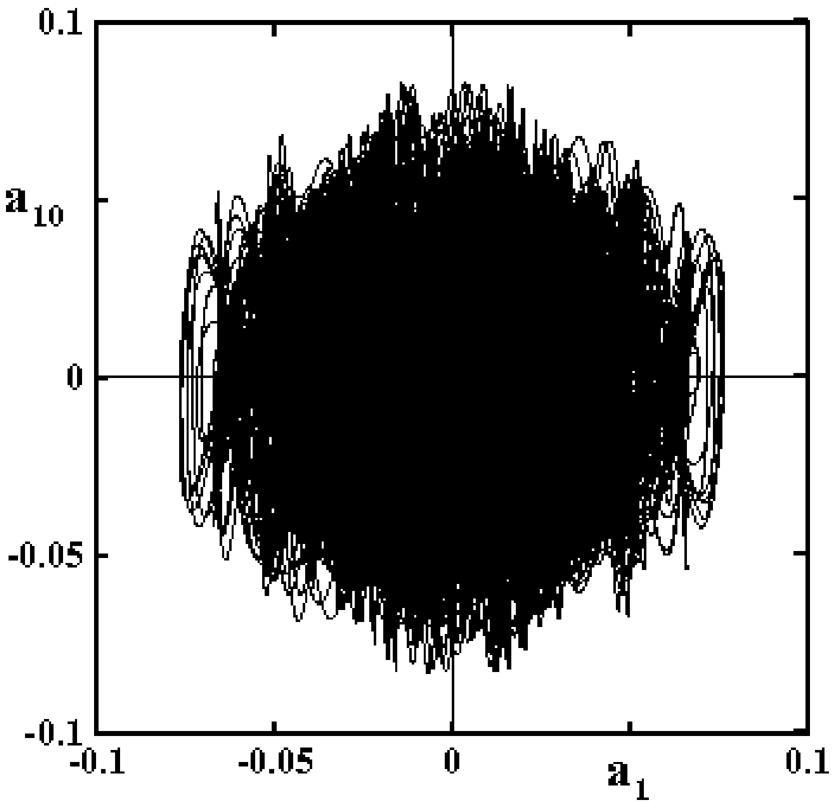
Good agreement between simulation • and FTT ○

Burgers' equation III

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

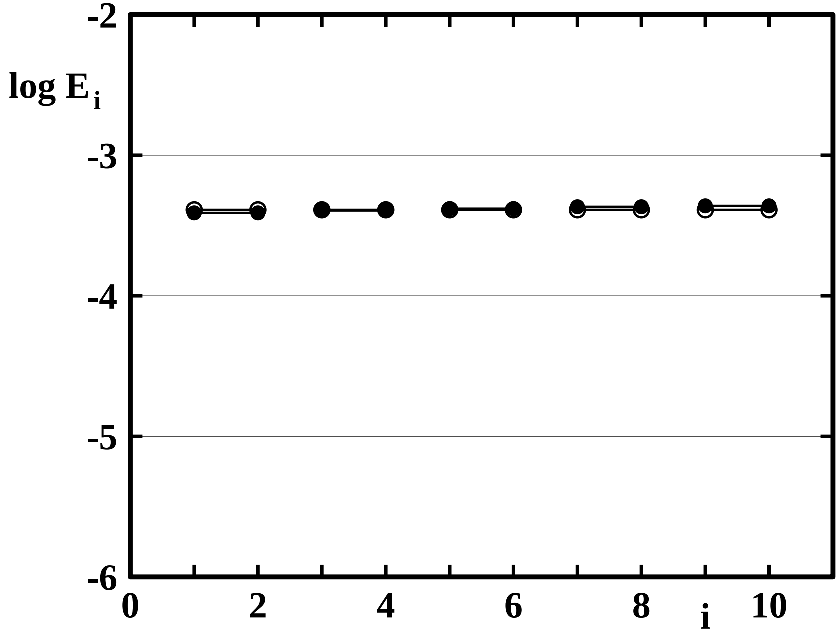
Truncated Burgers' solution **without** source and **without** diffusion term ≡ Majda & Timofeyev 2000

phase portrait



$U = 1,\ \sigma = 0,\ \nu = 0$

energy distribution



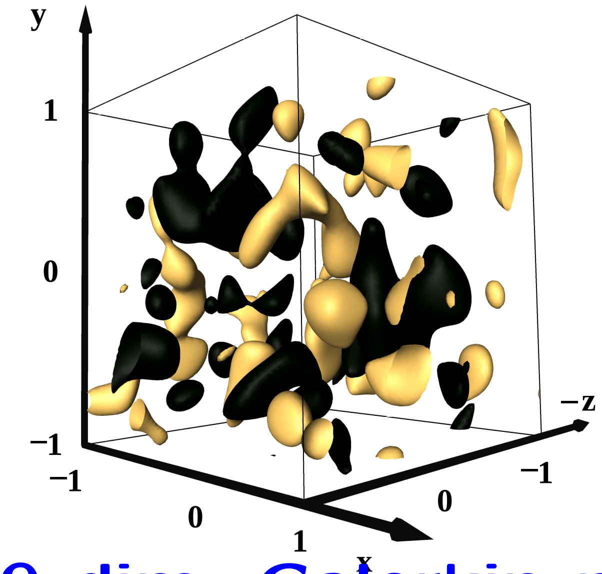
Equipartion of energy

in simulation • and FTT ○

Homogeneous shear turbulence ($Re = 1000$)

— ≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

3D flow



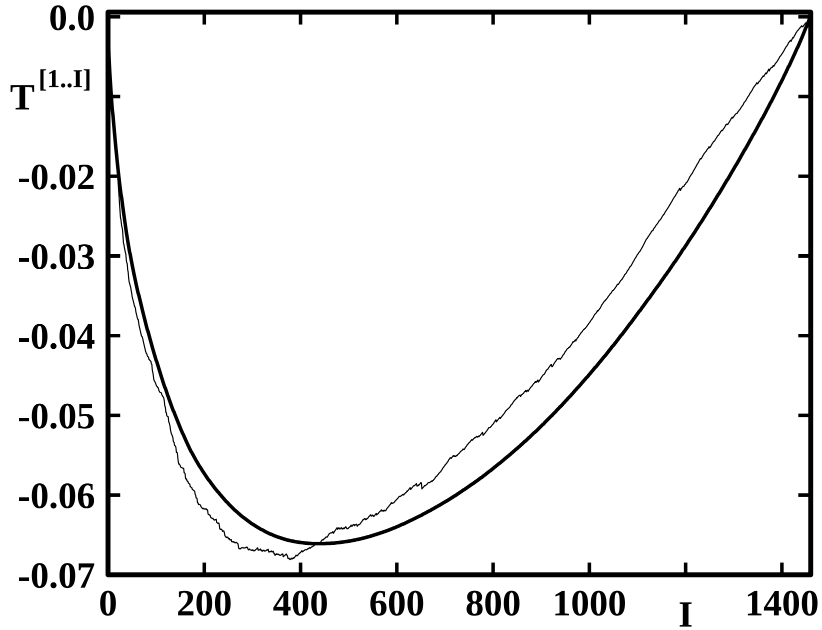
1459-dim. Galerkin model

$$\mathbf{u} = \sum_{i=0}^{N} a_i \mathbf{u}_i \quad \text{(Stokes modes)}$$

$$\dot{a}_i = c_i + \sum_{j=1}^{N} l_{ij}\, a_j + \sum_{j,k=1}^{N} q_{ijk}\, a_j\, a_k$$

Cumulative transfer term

(GM and FTT)



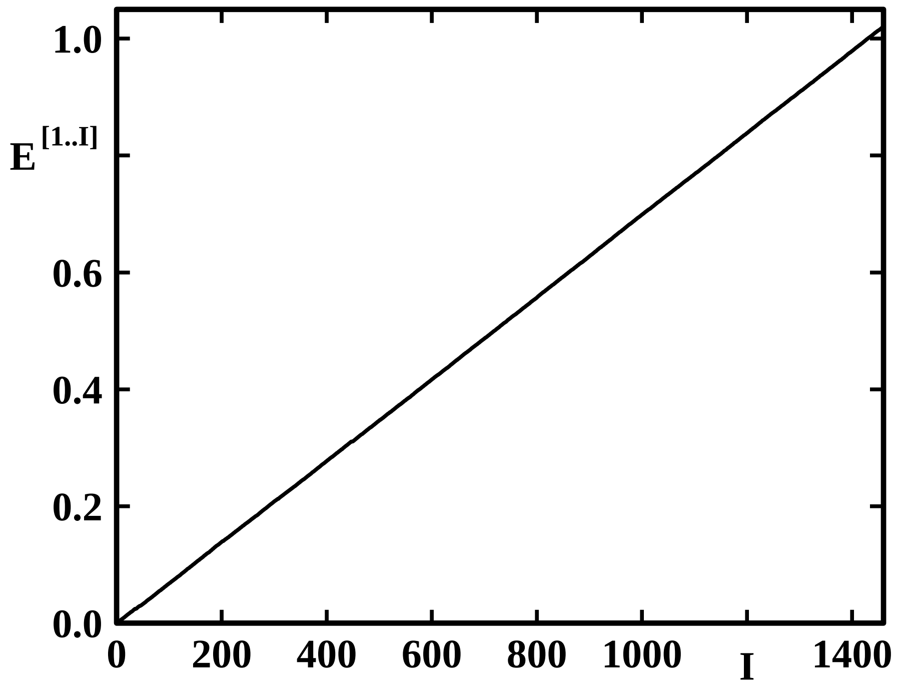
$$T^{[1..I]} := T_1 + \ldots + T_I$$

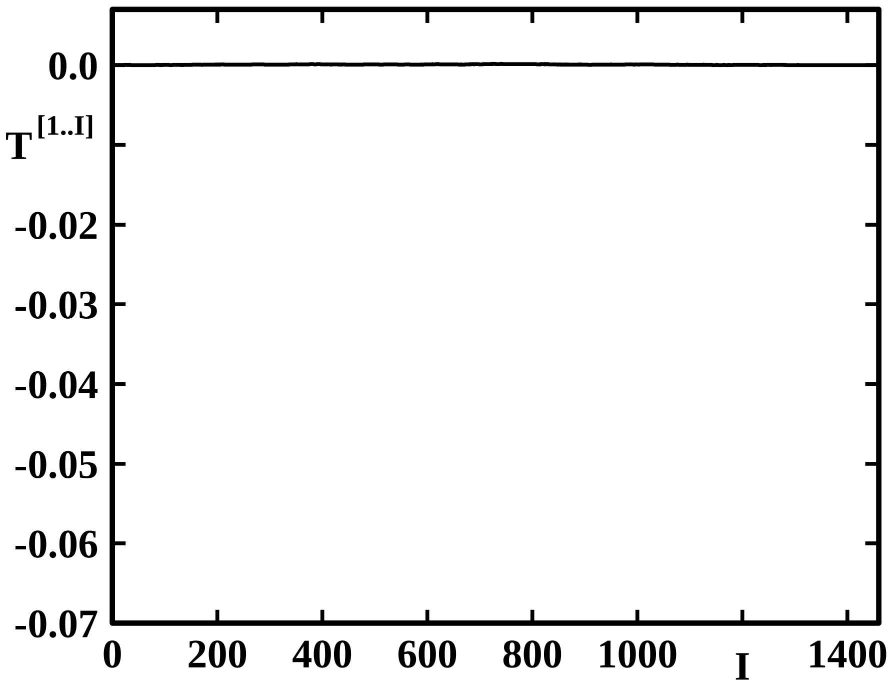
—: GM; — : FTT

Good agreement between GM and FTT prediction!

FTT modeling of truncated Euler solutions

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET*—





The systems approaches local thermal equilibrium ($E_1 = E_2 = \ldots = E_N$) without external interactions, i.e. $Q_i \equiv 0$.

FTT applications

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

— *and many follow-up publications* ≡ ≡ ≡ —

Instabilities and turbulence

- FTT generalizes the Landau equation $dA/dt = \sigma A - \beta A^3$
- rigorous system reduction of evolution equation

$$\mathbf{u}(\mathbf{x},t) = \sum_{i\in\mathcal{I}_{\text{td}}} a_i(t)\,\mathbf{u}_i(\mathbf{x}) + \sum_{i\in\mathcal{I}_{\text{mf}}} a_i(t)\,\mathbf{u}_i(\mathbf{x}) + \sum_{i\in\mathcal{I}_{\text{dyn}}} a_i(t)\,\mathbf{u}_i(\mathbf{x})$$

  - thermodynamic modes . . . . statistical treatment
  - mean-field (shift) modes . . . . algebraic equations
  - oscillatory modes . . . . dynamical system
- derivation of nonlinear subgrid turbulence model
- unified description of normal and inverse turbulence cascade

Variational principles

- statistical mechanics & definition of entropy
- MaxEnt principle for attractor

Attractor control

- E-based control $\mapsto$ manipulation of the turbulence cascade

$$\dot{a}_i = c_i + \sum_j c_{ij}\, a_j + \sum_{j,k} c_{ijk}\, a_j\, a_k + g_i\, b\,.$$

FTT for mean-field model

— ≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET —

Phase averaged GS

$$\frac{da_1}{dt} = \sigma\, a_1 - \omega\, a_2$$
$$\frac{da_2}{dt} = \sigma\, a_2 + \omega\, a_1$$
$$\frac{da_3}{dt} = \sigma_3\, a_3 + c\, \left(a_1^2 + a_2^2\right)$$

where $\sigma = \sigma_1 - \beta a_3$, $\omega = \omega_1 + \gamma a_3$,

$-\sigma_3 \gg \sigma_1$.

FTT generalization

$i = 1, 2 \quad \mapsto \quad$ thermodyn. mode
$i = 3 \quad \mapsto \quad$ mean-field mode

$i = 1$: $\quad \overline{a}_1 = 0 \quad E_1 \neq 0$
$i = 2$: $\quad \overline{a}_2 = 0 \quad E_2 \neq 0$
$i = 3$: $\quad \overline{a}_3 \neq 0 \quad E_3 = 0$

FTT equations

$$\frac{dE_1}{dt} = 2\sigma\, E_1 + \sum_{j,k=1}^{2} T_{1jk}$$
$$\frac{dE_2}{dt} = 2\sigma\, E_2 + \sum_{j,k=1}^{2} T_{2jk}$$
$$0 = \sigma_3\, a_3 + 2c\, (E_1 + E_2)$$

$\Rightarrow$ Watson-Stuart eqs.

Exploiting $E_1, E_2 \to E/2$:

$$\frac{dE}{dt} = 2\sigma\, E$$
$$0 = \sigma_3\, a_3 + 2c\, E$$

$\Rightarrow$ Landau equation

With $E := A^2/2$:

$$\frac{dA}{dt} = (\sigma_1 - \beta^\star A^2)\, A$$

FTT and statistical physics

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

	Maxwell-Boltzmann theory	**FTT**
entities	atoms	Galerkin modes
ensemble	micro ensemble	absolute equilibrium e.
coordinates	$\mathbf{u} = (u, v, w)$	$\mathbf{a} = (a_1, \ldots a_N)$
entropy	$S = -\int d\mathbf{u}\, p(\mathbf{u}) \log p(\mathbf{u})$	$S = -\int d\mathbf{a}\, p(\mathbf{a}) \log p(\mathbf{a})$
$S \overset{!}{=} \max$	Maxwell distribution $p(\Vert\mathbf{u}\Vert)$	analogous distribution $p(\mathbf{a}) = c\, e^{-\frac{a_1^2+\ldots+a_N^2}{\sigma^2}}$
equipartition principle	$E_u = E_v = E_w = kT/2$	$E_1 = \ldots = E_N = E/N$

In Maxwell-Boltzmann theory:

Mode coefficients a_i

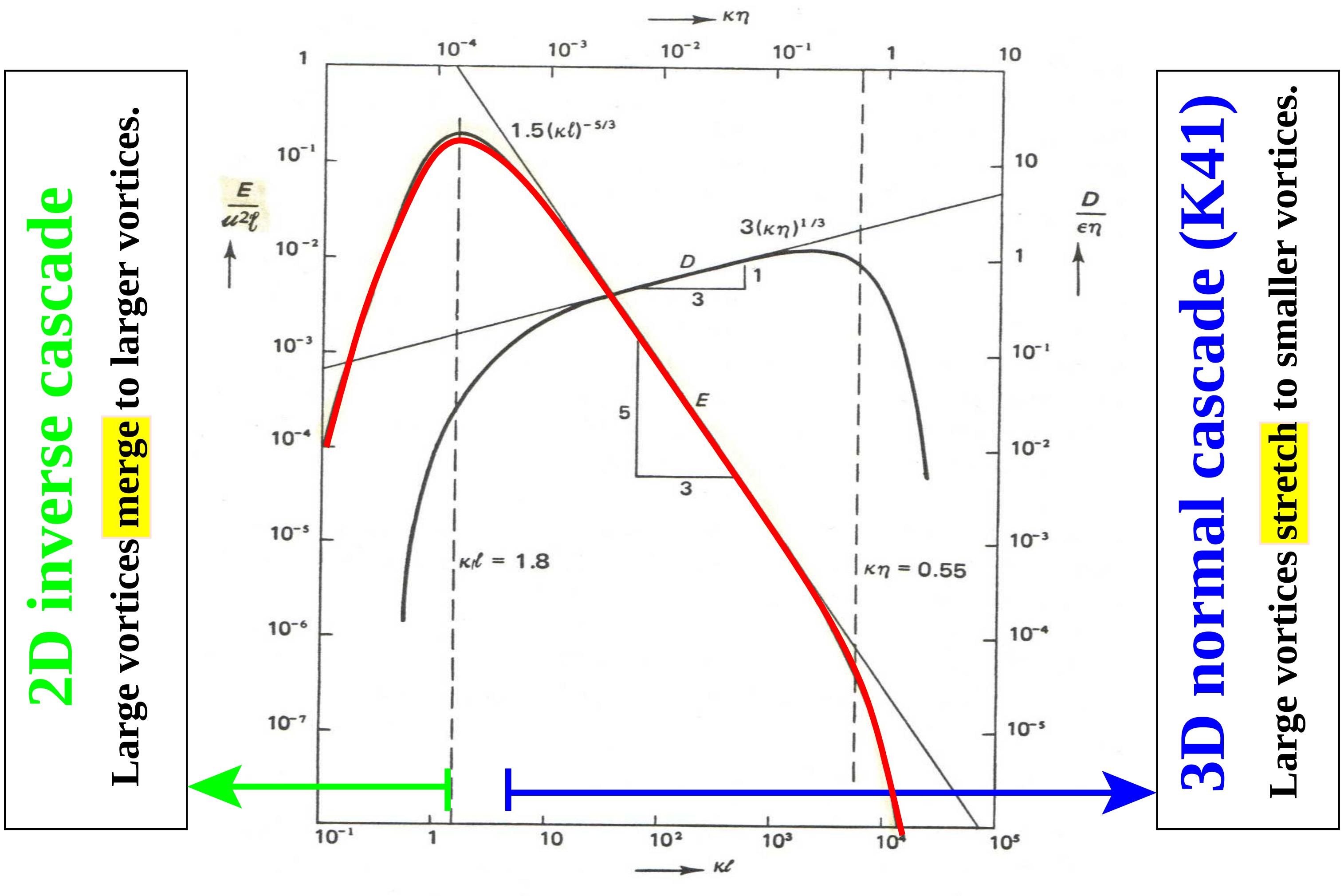
= generalized velocities!

James Clark Maxwell (1831–1879)

FTT and turbulence theory

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

In: Tennekes & Lumley 1972 Introduction to Turbulence, p. 270



Fick' laws for T_{ijk} explains normal and inverse cascade.

Energy flows downhill!

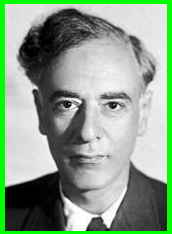
Overview

1. **Introduction**

   . . . . *physics & cybernetics dreams revisited*

2. **Mean-field modelling**

   . . . . *complete order / stability theory*

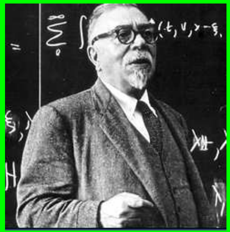


3. **Attractor modelling**

   . . . . *complete disorder / statistical physics*

4. **Attractor control**

   . . . . *Maxwellian and other deamons*

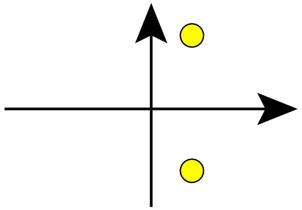


5. **Summary and outlook**

Dream #3: Control $\mapsto$ turbulence

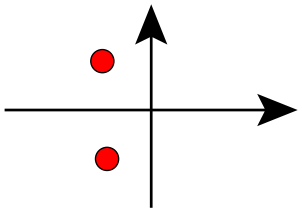
linear dynamics

$da/dt = A\,a + B\,b$



linear control

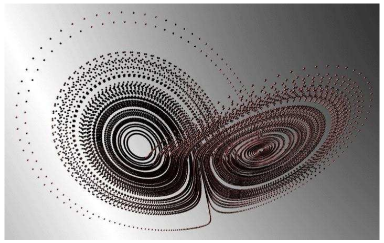
$b = K\,a$



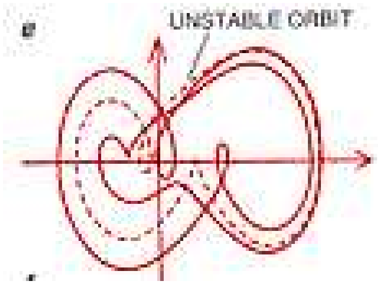
Anno Dazumal

strange attractor

$da/dt = f\,(a,b)$



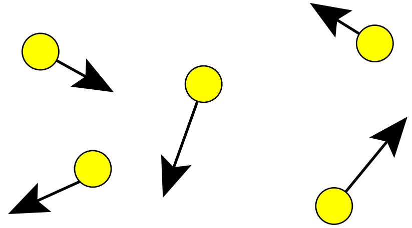
chaos control



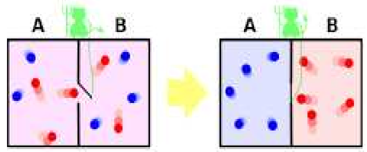
Ott, Grebogi, Yorke 1990 PRL

statistical physics

$S = k \ln W$



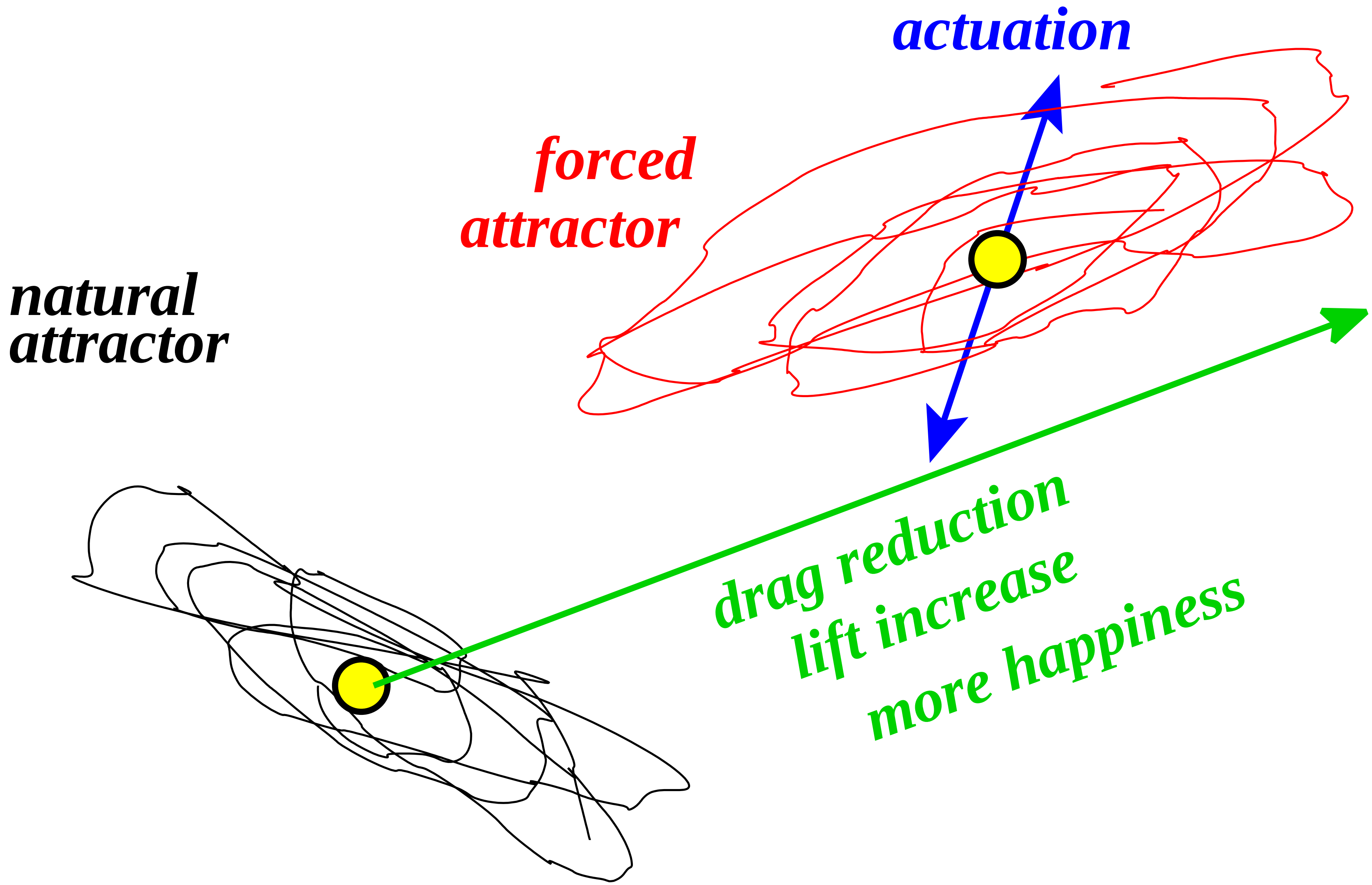
Maxwell's demon



Maxwell 1867
Wiener 1948

Turbulence control = attractor control

Phase space



Control design in FTT — an example

— ≡ *Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET* —

Goal functional (e.g. lift or drag) $Z = z_0 + \sum_{i=1}^{N} z_i \, \overline{a}_i$

Galerkin system (e.g. with single volume force)

$$\frac{da_i}{dt} = c_i + \sum_{j=1}^{N} l_{ij} a_j + \sum_{j,k=1}^{N} q_{ijk} a_j a_k + g_i \, b$$

Control ansatz $b = \sum_{i=1}^{N} k_i \, a_i$

Controlled dynamics with $l^c_{ij} = l_{ij} + g_i k_j$

$$\frac{da_i}{dt} = c_i + \sum_{j=1}^{N} l^c_{ij} a_j + \sum_{j,k=1}^{N} q_{ijk} a_j a_k$$

$\Rightarrow$ FTT applicable $\Rightarrow$ $\overline{a_i} = \overline{a_i}(k_1, \ldots, k_N)$

Control problem: $Z = Z(k_1, \ldots, k_N) = \max$

$\Rightarrow$ **Fully nonlinear, infinite horizon control!**

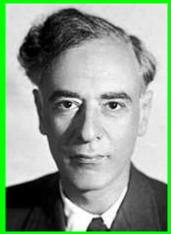
Overview

1. **Introduction**

   . . . . *physics & cybernetics dreams revisited*

2. **Mean-field modelling**

   . . . . *complete order / stability theory*

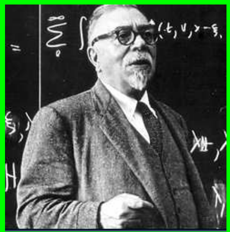


3. **Attractor modelling**

   . . . . *complete disorder / statistical physics*

4. **Attractor control**

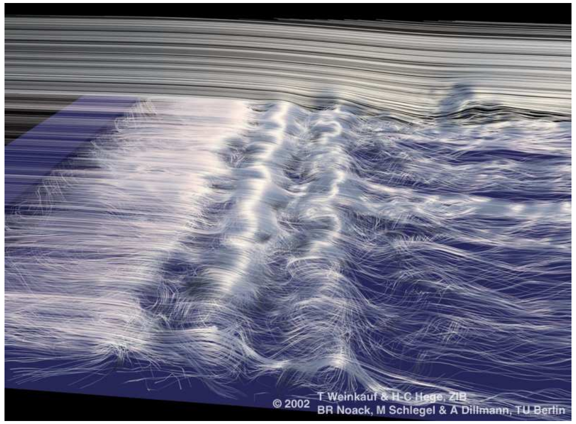
   . . . . *Maxwellian and other deamons*



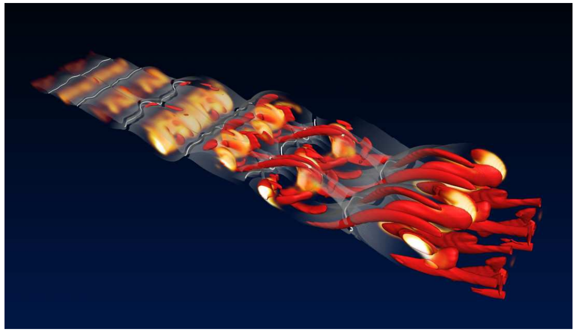
5. **Summary and outlook**

Configurations

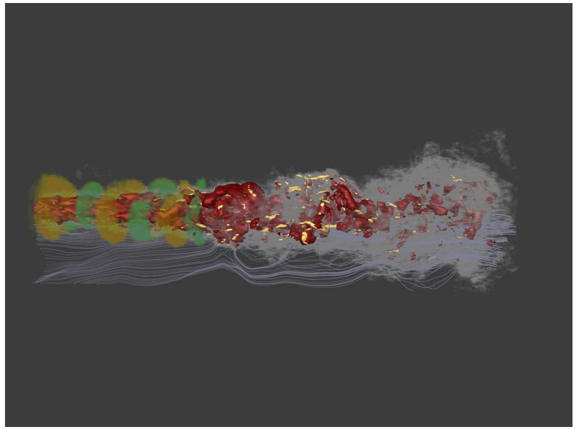
3D flow over a step



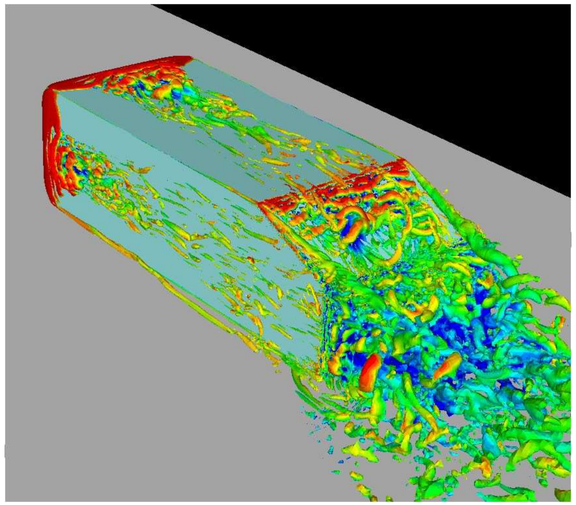
3D mixing layer



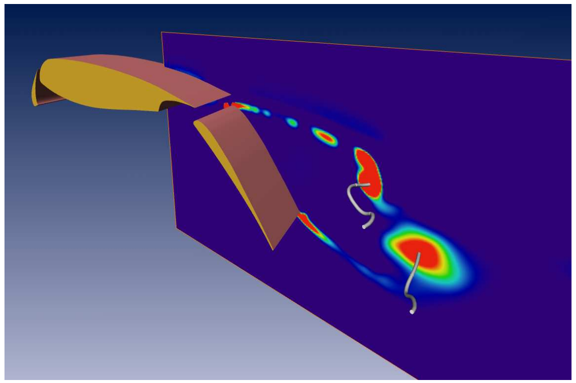
jet noise



Ahmed body



airfoil



wake
channel flow
combustor
cavity flow
. . .

Conclusions

- **Galerkin modelling** for **flow control** is a doable art!

$$\mathbf{u} = \sum_{i=0}^{N} a_i \, \mathbf{u}_i, \qquad \dot{a}_i = c_i + \sum_{j=1}^{N} l_{ij} \, a_j + \sum_{j,k=1}^{N} q_{ijk} \, a_j \, a_k + g_i \, b$$

http://BerndNoack.com

- **Turbulence control = attractor control**

Physics mechanisms are strongly nonlinear.

- drag reduction of D-shaped body
- lift increase of high-lift configuration
- ...

- **Model for natural and controlled attractor needed!**

$\Rightarrow$ **Upgrade Galerkin model with ergodic measure**

Conclusions

≡ Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET

- **Finite-time thermodynamics model** builds on GM

$$\mathbf{u} = \sum_{i=0}^{N} a_i \, \mathbf{u}_i, \qquad \dot{a}_i = c_i + \sum_{j=1}^{N} l_{ij} \, a_j + \sum_{j,k=1}^{N} q_{ijk} \, a_j \, a_k$$

$\Rightarrow$ **first and second moments** of unsteady flows

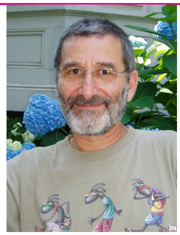
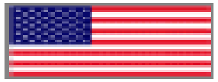
  - 1D Burgers' eq., • 2D wake, • 3D shear turbulence.

- **FTT $\mapsto$ Statistical physics (economics) link**
  - $\mathbf{u}_i$ ..... **person /thermodyn. degrees of freedom)**
  - E_i .... **wealth /order parameter**
  - $\sum l_{ij} a_j \Rightarrow Q_i$ ...... **pure capitalism /lin. instability**
  - $\sum q_{ijk} a_j a_k \Rightarrow T_i$ .... **pure communism /LTE**
  - **Both terms** .... **social market /partial LTE**

- **FTT $\mapsto$ energy-based and nonlinear control design**

More information

Call

 	 	 
+1-617-373.5277	+48-61-665.2778	+49-30-314.24732

... or read

- **Noack, Afanasiev, Morzyński, Tadmor & Thiele (2003) JFM** . . . . *generalized mean-field model* **EASY**
- **Noack, Papas & Monkewitz (2005) JFM** . . . . *modal energy flow analysis* **DIFFICULT**
- **Noack, Schlegel, Ahlborn, Mutschke, Morzyński, Comte & Tadmor (2008) JNET** . . . . *Finite-time thermodynamics* **INCOMPREHENSIBLE**

... or ask now!